Optimal Monetary Policy in an Estimated DSGE for the Euro Area*

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April 27, 2007

Abstract

The objective of this paper is to examine the main features of optimal monetary policy within a micro-founded macroeconometric framework. First, using Bayesian techniques, we estimate a medium scale closed economy DSGE for the euro area. Then, we study the properties of the Ramsey allocation through impulse response, variance decomposition and counterfactual analysis.

Welfare cost measures are also evaluated using an accurate second order approximation of the model. Finally, we show that relatively simple monetary policy rule can "approximate" the Ramsey allocation reasonably well. Such optimal simple operational rules seem to react specifically to nominal wage inflation. Overall, the Ramsey policy together with its simple rule approximations seem to deliver consistent policy messages and may constitute some useful benchmarks to conduct normative analysis within medium to large scale estimated DSGE framework.

Keywords: DSGE models, Monetary policy, Bayesian estimation, Welfare calculations.

JEL classification: E4, E5.

*First draft: March 2006.

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1 Introduction

The objective of this paper is to examine the main features of optimalx monetary policy within an empirically plausible micro-founded macroeconometric framework for the euro area. This paper contributes to the burgeoning literature related to the theory of monetary stabilization policy which investigates the design of optimal monetary policy and consider how such policy can be implemented.

The closed-economy medium-scale model we use is similar to the one estimated by Smets and Wouters [2003] which has been shown to account relatively well for euro area business cycles. The theoretical framework features both nominal frictions, with sticky prices and sticky wages, as well as real frictions with investment adjustment costs, variable capacity utilization, habit formation, and imperfect competition in product and labor markets. Economic fluctuations are driven by eight structural disturbances. Three efficient supply shocks are associated with technological progress, investment specific productivity and labor supply. Consumer preference and public expenditure disturbances constitute two efficient demand shocks. Time-varying labor income and firm revenue taxes generate price and wage markup shocks. Finally, we introduce an additional markup fluctuation related to the external finance premium. The typology of the structural disturbances embodied in the model is first guided by our objective to bring the theoretical model to the data but also reflects the need to analyze the optimal response to both efficient and inefficient, product and labor market shocks. In particular, the estimated residuals obtained from the econometric estimation of the first-order DSGE approximation will be used as structural sources of uncertainty to assess the stabilization properties of optimal policy. The limitations of such approach are two fold. In order to fit a relevant number of data, the range of shocks generally considered in the theoretical literature has to be extended, sometimes in directions which obviously lack sound micro-foundations. At the same time, alternative micro-foundation of disturbances can lead to observationally equivalent first-order DSGE approximation. In that case, the estimation strategy will not be able to identify in a decisive manner some source of fluctuations which can have crucially different normative properties. In this paper, we intend to illustrate those points by fully deriving the normative implications of the first-order estimation of the model.

Concerning the computation of the optimal policy, we solve the equilibrium conditions of the Ramsey allocation using second-order approximations to the policy functions. The numerical strategy is based on perturbation methods and is well-suited for our modelling framework, given the large number of state variables. This general method to derive the second-order approximation of the Ramsey solution allow us in principle to depart from some widespread restrictions used in the literature to rely on undistorted non-stochastic steady state. In addition, contrary to the linear-quadratic approach of Benigno and Woodford [2006] which approximates the Ramsey problem by linear quadratic one, the second-order approximation of the Ramsey allocation performed in this paper allows to depart from certainty equivalence and analyze the effect of policies on the first moment of the state variables. In the paper, since we intend to focus on the macroeconomic stabilization properties of the Ramsey policy in a medium-scale modelling framework, the constraint of efficient steady state is imposed to *ex ante* avoid creating additional policy tradeoffs due to the inefficient steady state and concentrate on the implications of the already rich structure of frictions and shocks on optimal policy.

The issue of implementing the Ramsey policy with an interest rate rule is addressed in the following way. First, a fully-fledged derivation of the robust interest rate rule in the sense of Giannoni and Woodford [2003a] is beyond the scope of this paper and would probably prove difficult to interpret given the number of state variable likely to enter the target criteria. Second, we restrict our attention to interest-rate rules which satisfy the following requirements. The interest rate is set as a function of a limited number of economic variables and concepts. We allow the model output gap to enter the feedback rule since its volatility has a strong impact on the welfare, and it remains a relevant economic concept for the stylized policy analysis pursued in this paper. In addition, the policy rule should induce a determinate equilibrium which satisfies the lower bound on nominal interest rate.

The most closely related paper to our study is Levin, Onatski, Williams, and Williams [2005] which examined the Ramsey allocation within an estimated DSGE on the US data and explored its implementation with simple rules. We share more specifically the inclusion in the normative analysis of a full set of disturbance processes. Such feature is of importance in our analysis since welfare computations and optimal simple rules that we provide in this paper, crucially depend on the structure of shocks and therefore should be computed with the appropriate exogenous sources of business cycles fluctuations. On this point we differ from Schmitt-Grohe and Uribe [2005] and Schmitt-Grohe and Uribe [2004] which only take into account three shocks.

The original contributions of our paper cover several dimensions. First, we make a special effort to illustrate the empirical properties of the Ramsey allocation for the euro area. Among the properties of the optimal monetary policy, we focus in particular on the driving factors the Ramsey allocation dynamics compared with the one derived from using the estimated interest rate rule. Obviously we first compare impulse response functions and variance decompositions for the historical rule and the Ramsey policy. This allows us to study the stabilization properties of the optimal policy across the different type of shocks. In addition, using counterfactual experiments based on the historical shocks for the euro area, we investigate the optimal policy reaction to fluctuations observed in the past and again emphasize the role of the different shocks in explaining the counterfactual dynamics.

A second novelty of our paper is that, unlike Levin et al. [2005], we incorporate the zero lower bound constraint into the analysis. We try to draw conclusions on the likelihood of occurrence of this constraint and more interestingly, on its normative implications. Our results indicate that contrary to what is shown in Schmitt-Grohe and Uribe [2005], the Ramsey policy is not operational in the sense that it induces a high probability to tilt the zero bound. This again points the importance of taking into account a full set of structural shocks. A more striking result is the negligible welfare cost of imposing the zero lower bound, meaning that even if the volatility of the policy instrument is highly constrained, monetary policy is still effective in improving the welfare of agents.

Third, the paper highlights the need to improve the economic micro-foundation and the econometric identification of the structural disturbances when bringing together estimated models and optimal policy analysis. In particular, we show that efficient labor supply shocks and inefficient wage markup shocks are close to observationally from the empirical perspective while they have crucially different implications for optimal policy. The labor supply shocks is indeed fully accommodated in the Ramsey allocation whereas the wage markup shocks are fully allowed to *pass-through* wage and price dynamics. Therefore, a better understanding of the labor market sources of fluctuation is required.

Finally, concerning the derivation of optimal simple rules, we try in this paper to propose an efficient computational technique. In order to approximate the Ramsey allocation with a simple interest-rate feedback rule, we compute the parameters of the rule by estimating the model on simulated data from the Ramsey allocation, using full information methods and constraining behavioral parameters as well as the stochastic properties of the structural shocks. This approach consists in finding the best simple rule in the sense of the marginal density of the simulated data while traditional approaches would find rules maximizing the welfare. Our method has the advantage of being much more efficient computationally and remains tractable with more sophisticated interest-rate rules. We show in particular that the optimal rule derived with this approach presents some similarities with the robust optimal rule in the sense of Giannoni and Woodford [2003a] which implement exactly the Ramsey allocation in a simplified model with price and wage stickiness. Moreover, we computed alternative simple operational interest-rate rule like Schmitt-Grohe and Uribe [2005]. Both exercises clearly indicated that such simple rules can relatively well approximate the Ramsey allocation but are crucially sensitive to the structure of economic shocks.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 describes the estimation and reports the results. Section 4 examines the welfare and dynamic properties of the optimal monetary policy. Section 5 considers the approximation of the optimal policy with simples instrument rules and standard policy preferences . Finally, section 6 concludes.

2 Theoretical model

The model is mainly based on Christiano, Eichenbaum, and Evans [2005] and Smets and Wouters [2003]. The sophistication of modelling framework is first guided by the need to match a certain level data coherence for the euro area, and in this respect, available studies point to an appropriate set of necessary frictions. However, we prefer to restrain this degree of sophistication in order to better understand the normative dimensions of the model, and in particular, we restrict our analysis to a closed economy set-up. Therefore, we introduce in the model some relevant frictions to induce intrinsic persistence in the propagation of shocks, including adjustment costs on investment and capacity utilization, habit persistence and staggered nominal wage and price contracts with partial indexation. In addition, we specify a sufficient number of structural shocks in order to account for the stochastic properties of the observed data.

Concerning policy evaluation, the needed second-order numerical approximation implies that the exact nonlinear recursive formulation of the complete set of equilibrium conditions should be derived. This is specifically relevant for the equilibrium Phillips curves for prices and wages as well as the micro-foundations of the associated markup shocks. Similarly, two additional variables which are constant at a first-order approximation, now appear in the non-linear setting, related to the measure of price and wage dispersion.

2.1 Households behavior

The economy is populated by a continuum of heterogenous infinitely living households. Each household is characterized by the quality of its labour services, $h \in [0,1]$. At time t, the intertemporal utility function of a generic household h is

$$\mathcal{W}_t(h) = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \varepsilon_{t+j}^B \left[\frac{\left(C_{t+j}(h) - \gamma C_{t+j-1}(h)\right)^{1-\sigma_c}}{1-\sigma_c} - \widetilde{L} \varepsilon_{t+j}^L \frac{L_{t+j}(h)^{1+\sigma_L}}{1+\sigma_l} \right] \tag{1}$$

Household h obtains utility from consumption of an aggregate index $C_t(h)$, relative to an internal habit depending on its past consumption, while receiving disutility from labor $L_t(h)$. Utility also incorporates a consumption preference shock ε_t^B and a labor supply shock ε_t^L. $\tilde{L}$ is a positive scale parameter.

Each household h wishes to maximize its intertemporal utility under the following budget constraint:

$$\begin{aligned} \frac{B_t(h)}{P_t R_t} + C_t(h) + I_t(h) = \frac{B_{t-1}(h)}{P_t} + \frac{\left(1-\tau_{w,t}\right) W_t(h) L_t(h) + A_t(h) + T_t(h)}{P_t} \\ + r_t^k u_t(h) K_{t-1}(h) - \Psi\left(u_t(h)\right) K_{t-1}(h) + \Pi_t(h) \end{aligned} \tag{2}$$

where P_t is an aggregate price index (see section), $R_t = 1 + i_t$ is the one period ahead nominal interest factor, $B_t(h)$ is a nominal bond, $I_t(h)$ is the investment level $W_t(h)$ is the nominal wage, $T_t(h)$ and $\tau_{W,t}$ are government transfers and time-varying labor tax, and

$$r_t^k u_t(h) K_{t-1}(h) - \Psi\left(u_t(h)\right) K_{t-1}(h)$$

represents the return on the real capital stock minus the cost associated with variations in the degree of capital utilization. The income from renting out capital services depends on the level of capital augmented for its utilization rate. The cost (or benefit) Ψ is an increasing function of capacity utilization and is zero at steady state, $\Psi(u^\star) = 0$. $\Pi_t(h)$ are the dividend emanating from monopolistically competitive intermediate firms. Finally $A_t(h)$ is a stream of income coming from state contingent securities and equating marginal utility of consumption across households $h \in [0,1]$. Separability of preferences ensures that households have identical consumption and investment plans.

2.1.1 Consumption choices

The first order condition related to consumption expenditures is given by

$$\lambda_t = \varepsilon_t^B \left(C_t - \gamma C_{t-1}\right)^{-\sigma_c} - \beta\gamma \mathbb{E}_t \left[\varepsilon_{t+1}^B \left(C_{t+1} - \gamma C_t\right)^{-\sigma_c}\right] \tag{3}$$

where λ_t is the lagrange multiplier associated with the budget constraint. The First order conditions corresponding to the demand for contingent bonds implies that

$$\lambda_t = R_t \beta \mathbb{E}_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}}\right] \tag{4}$$

Due to the assumed *internal* habit formation, the IS curve implied by the linearization of (3) and (4) is, in a sense, more forward looking than the one considered by Smets and Wouters [2003]: here consumption appears at lag one and leads one and two. As we are more interested in the normative implications of nominal rigidities, we choose an habit formation mechanism that does not generate by itself a distortion affecting the welfare.

2.2 Investment decisions

The capital is owned by households and rented out to the intermediate firms at a rental rate r_t^k. Households choose the capital stock, investment and the capacity utilization rate in order to maximize their intertemporal utility (1) subject to the intertemporal budget constraint (2) and the capital accumulation equation:

$$K_t = (1-\delta)K_{t-1} + \varepsilon_t^I \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{5}$$

where $\delta \in (0,1)$ is the depreciation rate, S is a non negative adjustment cost function such that $S(1) = 0$ and ε_t^I is an efficiency shock on the technology of capital accumulation.

This results in the following first order conditions:

$$Q_t = \mathbb{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \left(Q_{t+1}(1-\delta) + r^k_{t+1} u_{t+1} - \Psi\left(u_{t+1}\right) \right) \right] \varepsilon^Q_t \tag{6}$$

$$Q_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} S'\left(\frac{I_t}{I_{t-1}}\right) \right] \varepsilon^I_t + \beta \mathbb{E}_t \left[Q_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right) \varepsilon^I_{t+1} \right] = 1 \tag{7}$$

$$r^k_t = \Psi'\left(u_t\right) \tag{8}$$

We follow Smets and Wouters [2003] by introducing an *ad hoc* shock ε^Q_t accounting for fluctuations of the external finance risk premium.

2.2.1 Labor supply and wage setting

Each household is a monopoly supplier of a differentiated labour service. For the sake of simplicity, we assume that he sells his services to a perfectly competitive firm which transforms it into an aggregate labor input using a CES technology $L_t = \left[\int_0^1 L_t(h)^{\frac{1}{\mu_w}} \mathrm{d}h\right]^{\mu_w}$, where $\mu_w = \frac{\theta_w}{\theta_w - 1}$ and $\theta_w > 1$ is the elasticity of substitution between differentiated labor services. The household faces a labor demand curve with constant elasticity of substitution $L_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\frac{\mu_w}{\mu_w-1}} L_t$, where $W_t = \left(\int_0^1 W_t(h)^{\frac{1}{1-\mu_w}} \mathrm{d}h\right)^{1-\mu_w}$ is the aggregate wage rate.

Households set their wage on a staggered basis. Each period, any household faces a constant probability $1-\alpha_w$ of optimally adjusting its nominal wage, say $W^*_t(h)$, which will be the same for all suppliers of labor services. Otherwise, wages are indexed on past inflation and steady state inflation: $W_t(h) = \left[\pi_{t-1}\right]^{\xi_w} \left[\pi^\star\right]^{1-\xi_w} W_{t-1}(h)$ with $\pi_t = \frac{P_t}{P_{t-1}}$ the gross rate of (GDP) inflation. Taking into account that they might not be able to choose their nominal wage optimally in a near future, $W^*_t(h)$ is chosen to maximize the intertemporal utility (1) under the budget constraint (2) and the labor demand for wage setters unable to re-optimize after period t:

$$L_{t+j}(h) = \left(\frac{W^*_t(h)}{P_t}\right)^{-\frac{\mu_w}{\mu_w-1}} \left(\frac{P_t}{P_{t+j}} \left[\frac{P_{t-1+j}}{P_{t-1}}\right]^{\xi_w} \left[\pi^\star\right]^{j(1-\xi_w)} \right)^{-\frac{\mu_w}{\mu_w-1}} \left(\frac{W_{t+j}}{P_{t+j}}\right)^{\frac{\mu_w}{\mu_w-1}} L_{t+j}$$

The first order condition of this program can be written recursively as follows:

$$\frac{W^*_t(h)}{P_t} = \left(\mu_w \frac{\mathcal{H}^w_{1,t}}{\mathcal{H}^w_{2,t}} \right)^{\frac{\mu_w - 1}{\mu_w(1+\sigma_L)-1}} \tag{9a}$$

$$\mathcal{H}^w_{1,t} = \varepsilon^B_t \varepsilon^L_t \widetilde{L} L_t^{1+\sigma_L} w_t + \alpha_w \beta \mathbb{E}_t \left[\left(\frac{\pi_{t+1}}{\pi_t^{\xi_w} \left[\pi^\star\right]^{1-\xi_w}} \right)^{\frac{(1+\sigma_L)\mu_w}{\mu_w - 1}} \mathcal{H}^w_{1,t+1} \right] \tag{9b}$$

$$\mathcal{H}^w_{2,t} = (1-\tau_{w,t})\,\lambda_t L_t w_t^{\frac{\mu_w}{\mu_w-1}} + \alpha_w \beta \mathbb{E}_t \left[\left(\frac{\pi_{t+1}}{\pi_t^{\xi_w} \left[\pi^\star\right]^{1-\xi_w}} \right)^{\frac{1}{\mu_w - 1}} \mathcal{H}^w_{2,t+1} \right] \tag{9c}$$

where w_t denotes the real wage. Note that when wages are perfectly flexible (*ie* $\alpha_W = 0$), the wage setting scheme collapses to:

$$\frac{\mu_w}{(1-\tau_{w,t})} \varepsilon^B_t \varepsilon^L_t \tilde{L} L_t^{\sigma_l} = \lambda_t w_t$$

The real wage is equal to a markup $\frac{\mu_w}{1-\tau_{W,t}}$ over the marginal rate of substitution between consumption and labor. Finally, the aggregate wage dynamics is given by.

$$w_t^{\frac{1}{1-\mu_w}} = (1-\alpha_w)\left(\mu_w \frac{\mathcal{H}^w_{1,t}}{\mathcal{H}^w_{2,t}}\right)^{-\frac{1}{\mu_w(1+\sigma_l)-1}} + \alpha_w w_{t-1}^{\frac{1}{1-\mu_w}} \left(\frac{\pi_t}{\pi_{t-1}^{\xi_w} \bar{\pi}^{1-\xi_w}} \right)^{\frac{-1}{1-\mu_w}} \tag{10}$$

2.3 Producers behavior

2.3.1 Final good sector

Final producers are perfectly competitive firms producing an aggregate final good that may be used for consumption and investment. This production is obtained using a continuum of differentiated intermediate goods with the Dixit and Stiglitz [1977] production technology $Y_t = \left[\int_0^1 Y_t(z)^{\frac{1}{\mu_p}} \mathrm{d}z\right]^{\mu_p}$ where $\mu_p = \frac{\theta_p}{\theta_p - 1}$ and $\theta_p > 1$ is the elasticity of substitution between differentiated goods. The representative final good producer maximizes profits $P_t Y_t - \int_0^1 P_t(z) Y_t(z)\mathrm{d}z$ subject to the production function, taking as given the final good price P_t and the prices of all intermediate goods. The first order condition for this problem defines the factor demand function $Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\frac{\mu}{\mu-1}} Y_t$, $\forall z \in [0,1]$. Finally, as the sector is perfectly competitive, the zero profit condition holds and the expression for P_t is $P_t = \left[\int_0^1 P_t(z)^{\frac{1}{1-\mu}} \mathrm{d}z\right]^{1-\mu}$.

2.3.2 Intermediate firms

Firms, $z \in [0,1]$, are monopolistic competitors and produce differentiated products by using a common Cobb-Douglas technology:

$$Y_t(z) = \varepsilon^A_t \left(u_t K_{t-1}(z)\right)^\alpha L_t(z)^{1-\alpha} - \Omega$$

where ε_t^A is an exogenous productivity shock and $\Omega > 0$ is a fixed cost. A firm z hires its capital, $\widetilde{K}_t(z) = u_t K_{t-1}(z)$, and labor, $L_t(z)$, on a competitive market by minimizing its production cost. Given the real wage and rental rate of capital, the optimal behavior of firm z is to choose $\left(\widetilde{K}_t(z), L_t(z)\right)$ such that:

$$\frac{w_t L_t(z)}{r_t^k \widetilde{K}_t(z)} = \frac{1-\alpha}{\alpha} \quad \forall z \in [0,1]$$

the ratio of capital demand to labor demand is constant across firms. The real marginal cost is given by:

$$mc_t = \frac{w_t^{(1-\alpha)} \left[r_t^k\right]^{\alpha}}{\varepsilon_t^A \alpha^{\alpha} (1-\alpha)^{(1-\alpha)}} \tag{11}$$

The nominal profit of an intermediate firm z at time t is given by:

$$\Pi_t\big(P_t(z)\big) = \Big((1-\tau_{p,t})P_t(z) - P_t mc_t\Big) \left[\frac{P_t(z)}{P_t}\right]^{-\frac{\mu_p}{\mu_p - 1}} Y_t - P_t mc_t \Omega$$

where $\tau_{p,t}$ is a time varying tax on firm's revenue. In each period, a firm z faces a constant (across time and firms) probability $1 - \alpha_p$ of being able to re-optimize its nominal price, say $P_t^*(z)$. If a firm cannot re-optimize its price, the nominal price evolves according to the rule $P_t(z) = \pi_{t-1}^{\xi_p} \left[\pi^\star\right]^{(1-\xi_p)} P_{t-1}(z) \equiv \Gamma_{t,t-1} P_{t-1}(z)$, *ie* the nominal price is indexed on past inflation and steady state inflation. Let $\widetilde{\mathcal{V}}_t$ be the value at time t of an optimizing firm and $\mathcal{V}_t$ be the value at time t a non-optimizing firm. These values are defined as follows:

$$\widetilde{\mathcal{V}}_t = \max_{P_t^*} \left\{ \Pi\Big(\widetilde{P}_t(z)\Big) + \mathbb{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \Big((1-\alpha_p)\widetilde{\mathcal{V}}_{t+1} + \alpha_p \mathcal{V}_{t+1}\big(P_t^*\big)\Big)\right]\right\}$$

and

$$\mathcal{V}_t\big(P_{t-1}(z)\big) = \Pi\Big(\Gamma_{t,t-1}P_{t-1}(z)\Big) + \mathbb{E}_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \Big((1-\alpha_p)\widetilde{\mathcal{V}}_{t+1} + \alpha_p \mathcal{V}_{t+1}\big(\Gamma_{t,t-1}P_{t-1}(z)\big)\Big)\right]\Bigg\}$$

where λ_t is the lagrange multiplier associated to the household's budget constraint (2). The necessary and envelop conditions are given by:

$$(1-\theta_p)\left(\frac{P_t^*}{P_t}\right)^{-\theta_p} Y_t + \epsilon \left(\frac{P_t^*}{P_t}\right)^{-\theta_p - 1} mc_t Y_t + \alpha_p \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \mathcal{V}'_{t+1}\big(P_t^*\big)\right] = 0$$

and

$$\begin{aligned} \frac{\mathcal{V}'_t\big(P_{t-1}(z)\big)}{\Gamma_{t,t-1}} = (1-\theta_p)\left(\frac{\Gamma_{t,t-1}P_{t-1}(z)}{P_t}\right)^{-\theta_p} Y_t + \theta_p \left(\frac{\Gamma_{t,t-1}P_{t-1}(z)}{P_t}\right)^{-\theta_p - 1} mc_t Y_t \\ + \alpha_p \beta \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{P_t}{P_{t+1}} \mathcal{V}'_{t+1}\big(\Gamma_{t,t-1}P_{t-1}(z)\big)\right] \end{aligned}$$

Iterating on the envelop condition and substituting in the necessary condition we obtain[1]:

$$\frac{P_t^*}{P_t} = \mu_p \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha_p \beta)^j \lambda_{t+j} \left(\frac{\Gamma_{t+j,t}}{P_{t+j}/P_t}\right)^{-\theta_p} mc_{t+j} Y_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha_p \beta)^j \lambda_{t+j} \left(\frac{\Gamma_{t+j,t}}{P_{t+j}/P_t}\right)^{1-\theta_p} (1-\tau_{p,t+j})\, Y_{t+j}}$$

the optimal price of firm z relative to the aggregate price. Again, this condition can be written recursively:

$$\frac{P_t^*(z)}{P_t} = \mu_p \frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}} \tag{12a}$$

$$\mathcal{Z}_{1,t} = \lambda_t mc_t Y_t + \alpha_p \beta \mathbb{E}_t \left[\left(\frac{\pi_{t+1}}{\pi_t^{\xi_p} \left[\pi^\star\right]^{(1-\xi_p)}} \right)^{\frac{\mu_p}{\mu_p - 1}} \mathcal{Z}_{1,t+1} \right] \tag{12b}$$

$$\mathcal{Z}_{2,t} = (1-\tau_{p,t}) \lambda_t Y_t + \alpha_p \beta \mathbb{E}_t \left[\left(\frac{\pi_{t+1}}{\pi_t^{\xi_p} \left[\pi^\star\right]^{(1-\xi_p)}} \right)^{\frac{1}{\mu_p - 1}} \mathcal{Z}_{2,t+1} \right] \tag{12c}$$

As the distribution of prices among the share α_p of producers unable to re-optimize at t is similar to the one at $t-1$, the aggregate price index has the following dynamics:

$$P_t^{\frac{1}{1-\mu_p}} = \alpha_p \left(\pi_{t-1}^{\xi_p} \left[\pi^\star\right]^{1-\xi_p} P_{t-1} \right)^{\frac{1}{1-\mu_p}} + (1-\alpha_p) \left(P_t^*(z)\right)^{\frac{1}{1-\mu_p}}$$

or equivalently:

$$1 = \alpha_p \left(\frac{\pi_t}{\pi_{t-1}^{\xi_p} \left[\pi^\star\right]^{(1-\xi_p)}} \right)^{\frac{1}{\mu_p - 1}} + (1-\alpha_p) \left(\mu_p \frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}} \right)^{\frac{1}{1-\mu_p}} \tag{13}$$

When the probability of being able to change prices tends towards unity, (12) implies that the firm sets its price equal to a markup $\frac{\mu}{(1-\tau_{p,t})}$ over marginal cost.

2.4 Government

Public expenditures $G^\star$ are subject to random shocks ε_t^G. The government finances public spending with labor tax, product tax and lump-sum transfers:

$$P_t G^\star \varepsilon_t^G - \tau_{w,t} W_t L_t - \tau_{p,t} P_t Y_t - P_t T_t = 0 \tag{14}$$

[1]Where the cumulated gross price index is defined as

$$\Gamma_{t+j,t} = \Gamma_{t+1,t} \Gamma_{t+2,t+1} \dots \Gamma_{t+j,t+j-1} = \left[\pi^\star\right]^{j(1-\xi_p)} \left(\prod_{h=0}^{j-1} \pi_{t+h} \right)^{\xi_p}$$

2.5 Market clearing conditions

Aggregate demand is given by:

$$Y_t = C_t + I_t + G^\star \varepsilon_t^G + \Psi(u_t) K_{t-1} \tag{15}$$

where $K_t = \int_0^1 K_t(z)\mathrm{d}z$ is the aggregate demand of capital.

Market clearing condition on goods market is given by:

$$\begin{aligned}
\int_0^1 Y_t(z)dz &= \varepsilon_t^A \int_0^1 (u_t K_{t-1}(z))^\alpha (L_t(z))^{(1-\alpha)} dz - \Omega \\
&= \varepsilon_t^A u_t^\alpha \int_0^1 K_{t-1}(z) \left(\frac{L_t(z)}{K_{t-1}(z)}\right)^{(1-\alpha)} dz - \Omega \\
\Delta_{p,t} Y_t &= \varepsilon_t^A (u_t K_{t-1})^\alpha (L_t)^{1-\alpha} - \Omega
\end{aligned} \tag{16}$$

with $\Delta_{p,t} = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\frac{\mu}{\mu-1}} dz$ and L_t is the aggregate labor input.

$\Delta_{p,t}$ measures the price dispersion due to the staggered price setting. As in the case of the aggregate price index, we can show that this price dispersion index has the following dynamics:

$$\begin{aligned}
\Delta_{p,t} &= \alpha_p \int_0^1 \left(\frac{P_{t-1}(z)}{P_{t-1}} \frac{P_{t-1}}{P_t} \pi_{t-1}^{\xi_p} [\pi^\star]^{1-\xi_p}\right)^{-\frac{\mu}{\mu-1}} dz + (1-\alpha_p) \left(\frac{P_t^*(z)}{P_t}\right)^{-\frac{\mu}{\mu-1}} \\
&= \alpha_p \Delta_{p,t-1} \left(\frac{\pi_t}{\pi_{t-1}^{\xi_p} [\pi^\star]^{1-\xi_p}}\right)^{\frac{\mu}{\mu-1}} + (1-\alpha_p) \left(\mu \frac{\mathcal{Z}_{1,t}}{\mathcal{Z}_{2,t}}\right)^{-\frac{\mu}{\mu-1}}
\end{aligned} \tag{17}$$

The aggregate conditional welfare is defined by

$$\mathcal{W}_t = \int_0^1 \mathcal{W}_t(h) dh$$

We already mentioned that all household have the same consumption plans. Consequently, making use of the labor demand curve faced by each household we obtain

$$\mathcal{W}_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\tfrac{1}{1-\sigma_C} (C_{t+j} - \gamma C_{t-1+j})^{1-\sigma_C} - \tfrac{\varepsilon_{t+j}^L \tilde{L}}{1+\sigma_L} L_{t+j}^{1+\sigma_l} \Delta_{w,t+j} \right] \varepsilon_{t+j}^B$$

where we defined

$$\Delta_{w,t} = \int_0^1 \left(\frac{W_t(h)}{W_t}\right)^{-\frac{(1+\sigma_l)\mu_w}{\mu_w - 1}} dh$$

As for the price dispersion index, we can show that

$$\Delta_{w,t} = \alpha_w \Delta_{w,t-1} \left(\frac{w_t}{w_{t-1}} \frac{\pi_t}{\pi_{t-1}^{\xi_w} [\pi]^{\star 1-\xi_w}} \right)^{\frac{(1+\sigma_l)\mu_w}{\mu_w - 1}} + (1-\alpha_w)\, w_t \left(\mu_w \frac{\mathcal{H}^w_{1,t}}{\mathcal{H}^w_{2,t}} \right)^{-\frac{\mu_w(1+\sigma_l)}{\mu_w(1+\sigma_l)-1}}$$

2.6 Competitive equilibrium

Definition 2.1. *Given stochastic processes for the policy intrument, $\{R_t\}_{t=0}^{\infty}$, and shocks an equilibrium is a stochastic process for prices $\{\mathcal{P}_t\}_{t=0}^{\infty} = \{P_t, r_t^k, W_t, P_t(z), W_t(h)\,; h \in [0,1], z \in [0,1]\}_{t=0}^{\infty}$, and a stochastic process for quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty} = \{\{\mathcal{Q}_t^H\}_{t=0}^{\infty}, \{\mathcal{Q}_t^F\}_{t=0}^{\infty}\}$ with*

$$\begin{aligned}
\{\mathcal{Q}_t^H\}_{t=0}^{\infty} &= \{B_t, C_t, I_t, K_t, L_t, u_t, L_t(h); h \in [0,1]\}_{t=0}^{\infty} \\
\{\mathcal{Q}_t^F\}_{t=0}^{\infty} &= \left\{Y_t, \widetilde{K}_t(z), L_t(z), Y_t(z); z \in [0,1]\right\}_{t=0}^{\infty}
\end{aligned}$$

such that :

(i) Given a sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^H\}_{t=0}^{\infty}$ maximizes the representative agent lifetime utility;

(ii) Given a sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^F\}_{t=0}^{\infty}$ maximizes the representative firm profit;

(iii) For a sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty}$, the sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$ is such that all markets clear;

(iv) Prices satisfy (12a), (12b), (12c) and (13) ;

(v) Wages satisfy (9a), (9b), (9c) and (10).

3 Bayesian estimation of the linearized model

The exogenous shocks can be divided in three categories:

- Efficient shocks: shocks on technology, investment, labor supply (supply shocks), public expenditures and consumption preferences (demand shocks).
- Inefficient shocks: shocks on goods market markups, labor market markups, external risk premium.
- Policy shock: shock on the residual of the Taylor rule .

  Efficient shocks follow AR(1) processes whereas inefficient shocks and Taylor rule residual are white noises.

3.1 Data

We consider 7 key macro-economic quarterly time series from 1973q1 to 2004q4: output, consumption, investment, hours worked, real wages, GDP deflator inflation rate, and 3 month short-term interest rate. Euro area data are taken from Fagan et al (2001) and Eurostat. Concerning the euro area, employment numbers replace hours. Consequently, as in Smets and Wouters, hours are linked to the number of people employed e_t with the following dynamics:

$$e_t = \beta \mathbb{E}_t e_{t+1} + \frac{(1-\beta\alpha_e)(1-\alpha_e)}{\alpha_e}(l_t - e_t)$$

Aggregate real variables are expressed per capita by dividing with working age population. All the data are detrended before the estimation.

3.2 Parameters estimates

Some parameters are fixed prior to estimation. This concerns generally parameters driving the steady state values of the state variables for which the econometric model including detrended data is quasi uninformative. The discount factor β is calibrated to 0.99, which implies annual steady state real interest rates of 4%. The depreciation rate δ is equal to 0.0025 per quarter. Markups are 1.3 in the goods market and 1.5 in the labor market. The steady state is consistent with labor income share in total output of 70%. Shares of consumption and investment in total output are respectively 0.65 and 0.18.

All the results are obtained with Dynare, a matlab toolbox aimed at simulating and estimating DSGE models. The estimation strategy may be decomposed in three steps. First the linearized version of the rational expectation model is solved, so that the dynamics are described in a state-space representation (non linear in the deep parameters). Second, the posterior kernel of the model (i.e. the log-prior densities plus the log-likelihood of the model obtained by running a Kalman filter) is evaluated and maximized. Third, once the posterior mode is found, we get the entire posterior distribution by implementing a Metropolis-Hastings Monte-Carlo.

Regarding the prior distributions (see Table 1), the standard errors of the innovations are assumed to follow uniform distributions. In DSGE models, data are often very informative about the variance of structural disturbances so those very loose priors seem well suited. The distribution of the persistence parameters in the efficient and policy shocks is assumed to follow a beta distribution with mean 0.85 and standard error 0.1. Concerning the parameters of the Taylor rule, we follow Smets and Wouters [2003]: the long run coefficient on inflation and output gap are described by a Normal distribution with mean 1.5 and 0.125, and standard errors 0.1 and 0.05 respectively. The persistence parameter follows a normal around 0.75 with a standard error of 0.1. The prior on the short run reaction coefficients to inflation and output gap changes reflect the assumptions of a gradual adjustment towards the long run. Concerning preference parameters, the intertemporal elasticity of substitution is set at 1 with standard error of 0.375. The habit parameter is centered on 0.7 with standard deviation of 0.1 and the elasticity of labor supply has mean 2 and standard error of 0.75. Adjustment cost parameter for investment follows a $\mathcal{N}(4, 2)$ and the capacity utilization elasticity is set at 0.2 with a standard error of 0.1. Concerning the Calvo probabilities of price and wage settings, we assume a beta distribution around 0.75. The degree of indexation to past inflation is centered on 0.5.

Tab. 1 - PRIOR DISTRIBUTIONS

Parameter	Distribution	Mean	Std. dev.
σ_c	Normal	1.000	0.3750
σ_L	Normal	2.000	0.7500
γ	Beta	0.700	0.1000
α_p, α_w, α_e	Beta	0.750	0.0500
ξ_p, ξ_w	Beta	0.500	0.1500
ϕ	Gamma	0.200	0.1000
φ	Normal	4.000	0.5000
ψ_π	Normal	1.500	0.1000
$\psi_{\Delta\pi}$	Gamma	0.300	0.1000
ψ_y	Gamma	0.125	0.0500
$\psi_{\Delta y}$	Gamma	0.063	0.0500
ρ_i	Beta	0.750	0.1000
ρ_A, ρ_B, ρ_G, ρ_L, ρ_I	Beta	0.850	0.1000
σ_{ε^A} ,σ_{ε^P}, σ_{ε^W}	Uniform	2.000	1.1547
σ_{ε^B}, σ_{ε^L}, σ_{ε^Q}	Uniform	5.000	2.8868
σ_{ε^G}, σ_{ε^I}, σ_{ε^R}	Uniform	3.000	1.7321

Overall, the posterior distributions of the structural parameters are relatively similar to the one reported in Smets and Wouters [2003] even if our model specification is slightly different (see Table 2). In particular, we do not introduce a shock on the central bank inflation objective and the detrended output enters the Taylor instead of the model-based output gap. In addition, we consider here internal habits on consumption and not external habits since the latter would generate an addition source of non-Pareto optimality of the steady state. It is worth emphasizing that two parameters in particular are badly identified: the labor supply elasticity and the term on level inflation in the Taylor rule.

Tab. 2 - POSTERIOR DISTRIBUTIONS

Parameters	Post. mode	Post. mean	HPD inf	HPD sup
σ_c	1.9614	1.9591	1.5459	2.3997
σ_L	1.5027	1.8004	0.5957	3.0612
γ	0.4209	0.4366	0.3026	0.5546
α_p	0.9089	0.9098	0.8928	0.9260
α_w	0.7496	0.7660	0.7065	0.8292
α_e	0.8436	0.8448	0.8238	0.8659
ξ_p	0.2196	0.2434	0.1350	0.3491
ξ_w	0.2512	0.2624	0.1142	0.3975
φ	4.7521	4.8144	4.1001	5.5343
ϕ	0.7807	0.8279	0.4968	1.1433
ψ_π	1.5657	1.5762	1.4378	1.7163
$\psi_{\Delta\pi}$	0.2021	0.2015	0.1362	0.2609
ρ_i	0.8794	0.750	0.8551	0.9059
ψ_y	0.0970	0.125	0.0449	0.1591
$\psi_{\Delta y}$	0.2030	0.2033	0.1531	0.2494
ρ_A	0.9942	0.9861	0.9738	0.9991
ρ_B	0.8738	0.8666	0.7986	0.9376
ρ_G	0.9720	0.9633	0.9348	0.9906
ρ_L	0.9696	0.9591	0.9390	0.9798
ρ_I	0.9500	0.9325	0.8850	0.9790
σ_{ε^A}	0.5640	0.6001	0.5008	0.5008
σ_{ε^B}	2.1191	2.3108	1.6807	1.6807
σ_{ε^G}	1.8379	1.8551	1.6702	1.6702
σ_{ε^L}	3.7088	4.7944	1.9603	1.9603
σ_{ε^I}	1.0029	1.1090	0.7742	0.7742
σ_{ε^R}	0.1830	0.1860	0.1626	0.1626
σ_{ε^Q}	6.3809	6.4534	5.2116	5.2116
σ_{ε^P}	0.2826	0.2945	0.2574	0.2574
σ_{ε^W}	0.1949	0.2008	0.1656	0.1656

Regarding the estimation of the Taylor rule, we investigated the sensitivity of structural parameter estimates to diffuse priors and alternative specification (see Table 3). It turns out that the coefficient on the inflation level term in the policy rule is strongly affected by such changes. Parameter estimates other than the labor supply elasticity remain however broadly unchanged. In the rest of the paper, the estimated Taylor rule considered for comparison exercise is one on the benchmark model.

Tab. 3 - POSTERIOR PARAMETERS, SENSITIVITY TO MONETARY POLICY RULE SPECIFICATION

	Benchmark	Benchmark Diffuse priors	output gap	output gap Diffuse priors	Taylor	Taylor wage
ρ_A	0.994	0.993	0.993	0.997	0.975	0.984
ρ_B	0.874	0.863	0.755	0.768	0.390	0.386
ρ_G	0.972	0.968	0.952	0.939	0.972	0.979
ρ_L	0.970	0.972	0.959	0.971	0.951	0.964
ρ_I	0.950	0.935	0.954	0.945	0.906	0.907
φ	4.752	4.727	4.873	4.863	5.076	5.003
σ_C	1.961	1.901	1.618	1.632	1.185	1.205
h	0.421	0.427	0.628	0.623	0.879	0.854
α_W	0.750	0.751	0.693	0.681	0.731	0.680
σ_L	1.503	1.368	2.975	3.091	2.215	2.329
α_P	0.909	0.910	0.902	0.902	0.927	0.930
λ_E	0.844	0.843	0.838	0.844	0.843	0.855
ξ_W	0.251	0.250	0.294	0.297	0.229	0.250
ξ_P	0.220	0.225	0.297	0.282	0.224	0.218
ϕ	0.781	0.751	0.952	0.984	0.752	0.754
r_π	1.566	2.501	1.498	0.892	1.215	0.297
$r_{\Delta\pi}$	0.202	0.172	0.125	0.032	-	-
ρ	0.879	0.931	0.946	0.947	0.862	0.878
r_y	0.097	0.253	0.126	0.236	0.133	0.120
$r_{\Delta y}$	0.203	0.252	0.244	0.257	-	-
r_w	-	-	-	-	-	0.909
LDD	-467.452	-470.370	-463.371	-463.840	-502.054	-496.726

4 Ramsey approach to optimal monetary policy

4.1 Ramsey equilibrium

We define the Ramsey policy as the monetary policy under commitment which maximizes the intertemporal household's welfare.

Definition 4.1. *A Ramsey equilibrium is a competitive equilibrium such that :*

(i) Given a sequence of shocks, prices, policy instrument and quantities $\{\mathcal{P}_t, R_t, \mathcal{Q}_t\}_{t=0}^{\infty}$ maximize the representative agent lifetime utility given by

$$\mathcal{W}_o = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma_C} \left(C_t - \gamma C_{t-1} \right)^{1-\sigma_C} - \frac{\tilde{L} \varepsilon_{t+j}^{L}}{1+\sigma_L} L_t^{1+\sigma_L} \Delta_{w,t} \right] \varepsilon_t^B$$

(ii) $R_t > 1$

The Ramsey policy is therefore computed by formulating an infinite-horizon Lagrangian problem of maximizing the conditional expected social welfare subject to the full set of non-linear constraints forming the competitive equilibrium of the model. The first order conditions to this problem are obtained using symbolic Matlab procedures.

As it is common in the optimal monetary policy literature (see for example Khan, King, and Wolman [2003] and Schmitt-Grohe and Uribe [2005]), we assume a particular recursive formulation of the policy commitment labeled by Woodford [2003] as optimality *from a timeless perspective*. This imposes that the policy rule which is optimal in the latter periods is also optimal in the initial period and avoids the problem of finding initial conditions for the lagrange multipliers, which are now endogenous and given by their steady state values.

Since we are mainly interested in comparing the macroeconomic stabilization performances of different monetary policy regimes, we assume a fiscal intervention, namely subsidies on labor and goods markets, to offset the first order distortions caused by the presence of monopolistic competition in the markets. This ensure that the steady state is equivalent to the first best, and that the flexible price equilibrium is Pareto optimal.

The inequality constraint (ii) ensures that the zero lower bound (henceforth ZLB) on the nominal interest rate is not violated. Before going further into the optimal policy properties, we need to evaluate the quantitative relevance of this constraint. Let us define the deviation of interest rate from its steady state $\hat{R}_t = (R_t - R^\star)/R^\star$. Given that, in the steady state, $\beta = \pi^\star / R^\star$, the interest rate does not hit the ZLB if and only if:

$$(iii)\ \hat{R}_t > \beta/\pi^\star - 1$$

To assess the relevance of the ZLB we then simply need to examine the stationary distribution of $\hat{R}_t$ under different monetary policy regimes. Our results (see figure ZLB) indicate that the estimated rule implies a probability to tilt the zero bound of 13.7 percent under a zero steady state inflation rate, $\pi^\star = 1$, and 5 percent under a more reasonable two percent annual inflation rate, $\pi^\star = 1.005$. This result calls two comments. First, it highlights the only role left to the steady state inflation rate in our model, which is its effects on the ZLB constraint. Second, this first part of our exercise gives a rationale for a positive inflation target rate (here the steady state rate) in order to prevent central banks from hitting the ZLB.

Up to our knowledge, one of the few studies which tried to quantify the probability of hitting the zero bound in the euro area is Coenen [2003]. This paper reports, for an annual inflation rate of 2 percent, probabilities in a range of 2% to 17%, depending on specific modelling assumptions. Our estimates fall into this range, albeit closer to the lower bound.

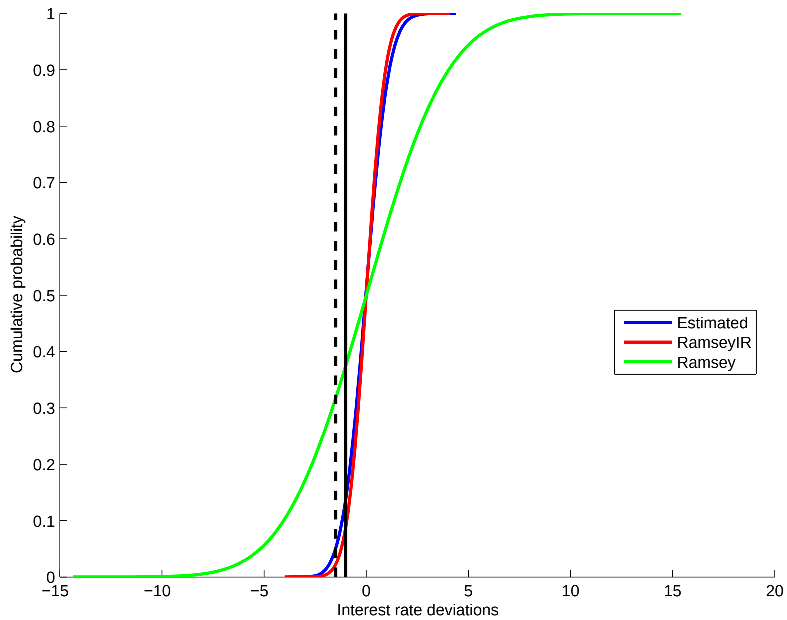
Now moving to the Ramsey policy, we find a probability to tilt the zero bound around 37.5 percent in a zero inflation steady state that only slightly falls to 31.7% under a 2% annual steady state inflation rate, making our Ramsey policy not operational. This sharply contrasts with recent results obtained by Schmitt-Grohe and Uribe [2005] within a medium scale macroeconomic model comparable to ours. They conclude that the low frequency of bindings of the ZLB makes it irrelevant as a constraint of the Ramsey problem. Where does this difference come from? It seems that the inclusion of rich structure of shocks, as opposed to three shocks, is the key point to understand this.

The ZLB is an occasionally binding constraint. To handle it, once need to resort on nonlinear global approximations solutions methods (see Christiano and Fisher [1997]). In a model like ours any attempt to use these methods is rendered impossible by the associated computational burden. To avoid high probabilities of hitting the zero bound under the Ramsey allocation, we thus follow Woodford [2003] by introducing in the households welfare a quadratic term penalizing the variance of the nominal interest rate:

$$\mathcal{W}_t^{IR} = \mathcal{W}_t + \lambda_r \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left(R_{t+j} - R^\star\right)^2 \tag{18}$$

where λ_r is the weight attached to the cost on nominal interest rate fluctuations. Instead of fixing this parameter to match a particular value of the probability to hit the zero bound, we

Fig. 1 - PROBABILITY TO HIT THE ZERO BOUND



Note: The straight and dashed black vertical lines plots *(iii)* for $\pi^{\star} = 1$ and $\pi^{\star} = 1.005$ respectively.

pragmatically choose to calibrate λ_r so that, under the Ramsey policy, the unconditional variance of the nominal interest rate is close to the historical one. Under this assumption, the probability to hit the zero bound is now reasonably low, for both $\pi^\star = 1$ and $\pi^\star = 1.005$.

Therefore, in order to make the Ramsey operational, we constrain the volatility of the policy instrument. Does it mean that the zero bound really limit the economic effectiveness of monetary policy? The following section investigates the property of the constrained Ramsey allocation. Unless Otherwise indicated, our results are computed at the mode of the estimated posterior distributions.

4.2 Comparison of the constrained and unconstrained Ramsey allocation

In order to investigate the implications of the additional welfare penalization for interest rate fluctuations on the optimal policy, we first compare the welfare costs of both policies, using conditional welfare on the steady state Ramsey allocation. More specifically, we compute the fraction of consumption stream from alternative monetary policy regime to be added (or subtracted) to achieve the reference level corresponding to the allocation following the estimated policy rule. That is, we measure the welfare cost in percentage points, $welfarecost = \psi \times 100$, by solving for ψ the following equation,

$$\mathcal{W}_t^{est} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{1-\sigma_C} \left(C_{t+j}^a - \gamma C_{t-1+j}^a \right)^{1-\sigma_c} (1+\psi)^{1-\sigma_c} - \frac{\tilde{L} \varepsilon_{t+j}^L}{1+\sigma_L} L_{t+j}^{a(1+\sigma_l)} \Delta_{W,t+j}^a \right] \varepsilon_{t+j}^B$$

which gives

$$\psi = \left[\frac{\mathcal{W}_t^{est} + \mathcal{W}_{t,L}^a}{\mathcal{W}_t^a + \mathcal{W}_{t,L}^a} \right]^{\frac{1}{1-\sigma_c}} - 1$$

where $\mathcal{W}_t^{est}$ is the welfare obtained under the estimated policy rule, X_t^a denotes the variable X_t under the alternative policy regime and $\mathcal{W}_{t,L}^a = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\tilde{L} \varepsilon_{t+j}^L}{1+\sigma_L} L_{t+j}^{a(1+\sigma_L)} \Delta_{W,t+j}^a$.

Table 4 reports welfare cost measures relative to the estimated rule, where the non constrained optimal policy is referred as the Ramsey while the operational one is referred as the RamseyIR. First, we can observe that the welfare costs are similar with the Ramsey or RamseyIR, amounting respectively to 2.15 and 2.14 percent loss in consumption each period. Therefore, even if the volatility of the policy instrument is highly constrained, monetary policy is still effective in improving the welfare of agents. This point is even more sensible when we compare the distribution of the welfare costs drawn from the posterior distribution of the parameters (see Figure 14). The distribution of the Ramsey and RamseyIR welfare costs are almost identical.

Turning to second order moments, Table 4 shows that the penalization for interest rate volatility in the welfare function is not affecting strongly the variance of output components and inflation in the optimal allocation. The same conclusion will hold by analyzing the respective impulse responses and variance decompositions under both policy regimes. Consequently, the operational feature that we implemented in the Ramsey allocation is sufficient to maintain the fluctuations of the policy rates within reasonable range but does not deteriorate significantly the stabilization properties of the optimal policy. In the following sections, the Ramsey policy will refer to the optimal allocation derived by using the modified welfare function and will be compared with the estimated rule across several dimensions.

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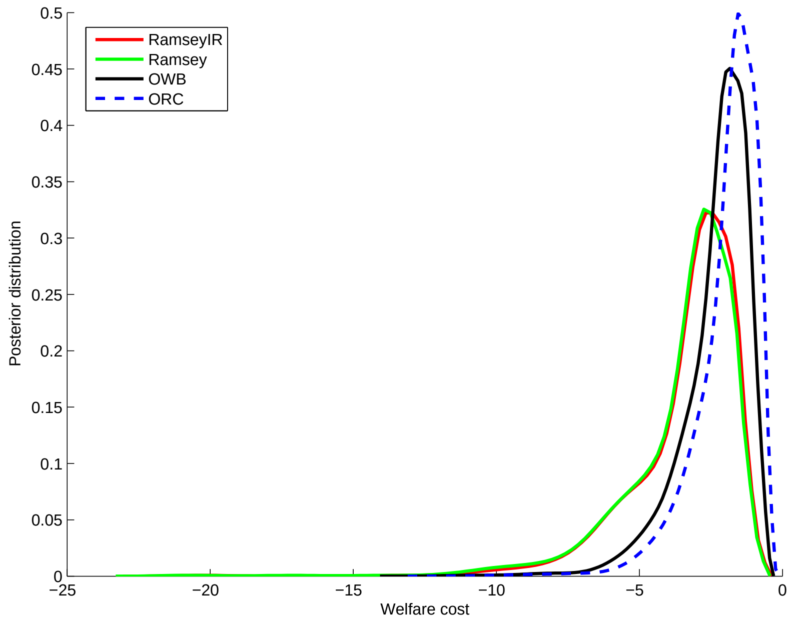
4.3 Welfare Cost and second order moments

As mentioned in the previous section, the conditional welfare gain of the optimal policy compared with the estimated rule is around 2.1 percentage point of consumption (see Table 4). Such gain is even higher when measured by unconditional welfare. By construction, the unconditional welfare measure is a weighted average of the conditional welfare levels associated with all possible values of the state vector with weights given by their unconditional probabilities. With this measure, the gain of optimal policy over the estimated rules averages 3.5%. In terms of volatility of macroeconomic aggregates, the Ramsey allocation allows for more fluctuations in real quantities while the variations of inflation and nominal wage growth are much more muted than with the estimated rule. Finally, the welfare gains of the Ramsey allocation are also illustrated by the higher unconditional mean levels of both real and nominal variables.

Tab. 4 - SELECTED SECOND ORDER MOMENTS

	Estimated	Ramsey	RamseyIR
Std. dev.			
Output	5.26	7.26	7.25
Consumption	6.28	7.61	7.59
Investment	12.27	17.42	17.44
Wage Inflation	1.11	0.29	0.32
Inflation	0.97	0.27	0.27
Interest Rate	0.91	3.13	0.74
Stochastic Steady State Deviations			
Output	3.50	5.31	5.42
Consumption	2.13	4.39	4.52
Investment	20.97	23.18	23.34
Wage Inflation	-0.61	0.02	0.02
Inflation	-0.61	0.02	0.02
Interest Rate	-0.61	-0.04	-0.01
Welfare			
Cond. cost	0	-2.15	-2.14
Uncond. cost	0	-3.49	-3.51
Cond. level	-173.35	-170.92	-170.93
Uncond. level	-171.27	-167.34	-167.32

Fig. 2 - WELFARE COST POSTERIOR DISTRIBUTIONS



The results presented in Table 4 are obtained assuming subsidies in product and labor markets as well as steady state markups of 1.3 and 1.5 respectively. Table 5 and Table 6 analyze the implications of removing the subsidies and lowering the markups. The welfare costs increase without the subsidies, augmenting by 0.6 percentage point in the benchmark case. Standard deviations of real and nominal variable are slightly higher across both policies when the subsidies are removed. Regarding the stochastic steady state, the unconditional mean levels of inflation and wage growth are left quasi unchanged while the mean levels of real variable are substantially higher for the optimal policy without subsidies.

4.4 Impulse responses analysis

The dynamics of the Ramsey allocation is computed by solving the first-order approximation of the equilibrium conditions. Figures 2 to 9 show the median impulse response functions and the 90% highest posterior IRF density interval for the estimated Taylor rule and the Ramsey policy.

Regarding productivity shock, the Ramsey allocation generates a stronger and faster response of real variables and real wage while the downward pressures on prices are much more

Tab. 5 - SENSITIVITY ANALYSIS 1

	Welfare Cost	Welfare Level	Std. dev. Output	Std. dev. Inflation	Std. dev. Wage Inflation
Subsidies					
Estimated					
benchmark	0	-173.35	5.26	0.97	1.11
$\mu = 1.1$	0	-176.09	5.11	0.96	1.09
$\mu_w = 1.3$	0	-173.90	5.04	0.94	1.04
RamseyIR					
benchmark	-2.14	-170.93	7.25	0.27	0.32
$\mu = 1.1$	-4.17	-171.37	7.31	0.26	0.34
$\mu_w = 1.3$	-2.50	-171.07	7.26	0.28	0.31
No Subsidies					
Estimated					
benchmark	0	-219.57	5.38	1.02	1.14
$\mu = 1.1$	0	-178.25	5.21	0.99	1.11
$\mu_w = 1.3$	0	-228.10	5.3	1	1.09
RamseyIR					
benchmark	-2.77	-215.07	8.15	0.28	0.40
$\mu = 1.1$	-4.11	-172.95	7.60	0.27	0.37
$\mu_w = 1.3$	-3.25	-222.83	8.11	0.9	0.35

Tab. 6 - SENSITIVITY ANALYSIS 2

	Stochastic Steady State Deviations (*in percentage*)				
	Output	Consumption	Investment	Inflation	Wage Inflation
Subsidies					
Estimated					
benchmark	3.50	2.13	20.97	-0.61	-0.61
$\mu = 1.1$	2.18	0.10	20.53	-0.38	-0.38
$\mu_w = 1.3$	3.76	2.48	21.16	-0.65	-0.65
RamseyIR					
benchmark	5.42	4.52	23.34	0.02	0.02
$\mu = 1.1$	5.24	4.27	23.23	0.07	0.07
$\mu_w = 1.3$	5.96	5.22	23.89	0.02	0.02
No Subsidies					
Estimated					
benchmark	3.60	2.64	20.01	-0.62	-0.62
$\mu = 1.1$	2.17	0.26	20.07	-0.38	-0.38
$\mu_w = 1.3$	3.84	2.99	20.14	-0.66	-0.66
RamseyIR					
benchmark	6.70	6.25	25	0.02	0.02
$\mu = 1.1$	5.82	4.99	24.07	0.07	0.07
$\mu_w = 1.3$	7.21	7	25.23	0.01	0.01

limited. The associated interest rate path is more accommodative in the short term but reverts very rapidly to its initial level. Notice that over longer horizons, the response of real variables becomes significantly closer in both monetary regimes. The other efficient supply shock in the model is the labor supply shock for which the differences highlighted above turn out to be even more pronounced. The timely and hump-shaped decrease in interest rate under the Ramsey policy stimulates output, consumption and investment while leaving quasi unchanged inflation and real wages. By contrast, the estimated rule is not supportive enough to prevent a decrease in real wage and inflation.

Turning to efficient demand shocks, the Ramsey policy leans against preference shocks. The increase in consumption is more limited than under the estimated rule and the contraction in investment is stronger. Overall, GDP decreases in short term under the Ramsey policy while inflation and real wages are almost fully stabilized. Under the estimated rule, the preference shock is expansionary on GDP and upward pressures emerge on real wages and inflation. Differences are less pronounced for the other shocks affecting demand components. The responses of GDP, consumption, investment and real wages to an investment shock or a government spending shock are relatively similar under the Ramsey policy and the estimated rule. However the inflation response is much more muted in the Ramsey allocation.

Considering inefficient shocks, the transmission of price markup shocks to the economy is not strongly different under both monetary regimes, suggesting similar inflation (prices and wage)/output tradeoff for this type of shock. However, in the case of wage markup shocks and external finance premium shocks, the Ramsey policy is much more restrictive, delivering lower real variables and more stable inflation.

Overall, compared with the estimated Taylor rule, the Ramsey policy accommodates more strongly the efficient supply shocks, leans more against efficient demand shocks, and in the case of markup shocks, tilts the inflation/output tradeoff towards inflation stabilization. In addition, the optimal policy is much more responsive to labor market shocks than the estimated rule which incorporates only goods market variables such as inflation and output.

4.5 Variance decomposition

Turning now to the contribution the various structural shocks to the variance of forecast errors, the comparison between the results obtained under the estimated rule and the one associated with the Ramsey allocation confirms the properties identified above (see Table 7).

Regarding activity, the contribution of efficient supply shocks to the variance of forecast errors on output is much higher under the Ramsey policy over the short to medium term. In particular, the labor supply shock accounts for around 75% of the forecast errors at a two years horizon under the Ramsey policy, compared with less than 5% under the estimated rule. Conversely, demand shocks, price markup shocks and equity premium shocks contribute more strongly with the estimated rule up to a two years horizon. Wage markup shocks have a stronger impact of forecast errors in the Ramsey allocation.

The Ramsey policy is significantly muting the impact of efficient shocks on inflation forecast errors. While efficient supply shocks account for 90% of inflation variance in the long run under the estimated rule, this share is reduced to less than 10% under the Ramsey policy. Price markup shocks are the main source of forecast errors in the very short term with the estimated rule but its contribution rapidly decreases at longer horizon. Under the Ramsey policy, price markup shocks explain more than 90% of forecast errors at all horizon.

Concerning interest rates, efficient supply shocks contribute relatively more to the variance of forecast errors under the estimated rule, at all horizon. Efficient demand shocks contribute more in the short run but less in the long run under the estimated rule. The main difference regards the wage markup shocks which explains more than 40% of forecast errors in the medium term under the Ramsey policy, compared with less than 1% under the estimated rule.

4.6 Counterfactual analysis

Moreover, the particular features of the optimal policy highlighted previously can be illustrated in terms of counterfactuals (see Figures 11 to 14). Given the estimated structural shocks, we simulate the path of the main macroeconomic aggregates under the Ramsey policy. Overall, the optimal policy would have implied higher GDP growth in the mid-80's and in the mid-90's but lower growth around 1990 and 2000. The dynamics of consumption would not have been significantly affected and investment would have accounted for most of the GDP growth differences. Inflation and the model-based output gap would have been much more stable under the Ramsey policy. This outcome would have been achieved with a path of the policy qualitatively similar to the observed one but with a higher amplitude of the policy changes. Notice that the strong performance of the Ramsey policy in terms of output gap and inflation comes at a limited cost regarding the policy rate volatility. Let us now turn to a comparison of the contribution of historical shocks to activity, inflation and interest rate under the Ramsey

Tab. 7 - COMPARISON OF VARIANCE DECOMPOSITION

	Estimated				Ramsey			
Quarters	0	4	8	∞	0	4	8	∞
Output								
ε^A	1.32	0.71	4.17	61.03	3.69	8.80	11.20	47.82
ε^L	0	0.45	4.66	25.27	61.56	72.02	73.04	46.50
ε^I	0.11	3.17	7.9	3.47	0.82	2.18	2.70	1.42
ε^B	30.86	30.98	22.08	2.14	0.16	4.01	4.63	1.51
ε^G	20.65	8.6	6.52	1.44	15.11	3.71	2.63	1.24
ε^R	20.47	39.33	41.22	5.21	-	-	-	-
ε^Q	22	10.51	7.48	0.74	4.50	0.49	0.27	0.06
ε^P	4.01	6.08	5.83	0.67	2.00	1.73	1.56	0.51
ε^W	0.59	0.17	0.13	0.03	12.15	7.06	3.98	0.95
Inflation								
ε^A	13.08	29.92	32.72	60.59	1.74	6.58	7.55	7.80
ε^L	14.48	37.72	43.26	30.17	0.07	0.28	0.28	0.56
ε^I	0	0.01	0.01	0.09	0.00	0.00	0.02	0.13
ε^B	1.14	2.64	2.62	1.12	0.00	0.00	0.01	0.06
ε^G	0.05	0.14	0.16	0.09	0.01	0.02	0.02	0.02
ε^R	0.99	2.41	2.51	1.15	-	-	-	-
ε^Q	0.02	0.04	0.03	0.02	0.00	0.01	0.01	0.01
ε^P	70.05	26.87	18.49	6.71	98.15	93.06	92.06	91.36
ε^W	0.19	0.26	0.2	0.07	0.03	0.05	0.05	0.06
Interest Rate								
ε^A	4.42	16.88	24.48	63.57	1.64	2.51	2.49	2.67
ε^L	2.10	20.25	32.56	25.15	7.89	14.03	14.51	15.45
ε^I	0.05	1.85	3.67	2.43	0.19	0.20	0.52	2.43
ε^B	16.61	26.27	19.21	4.45	8.28	24.72	28.06	27.19
ε^G	8.58	3.71	2.12	0.49	0.28	0.50	0.53	0.63
ε^R	54.66	23.17	13.12	2.86	-	-	-	-
ε^Q	9.97	5.98	3.70	0.79	10.61	8.68	8.20	7.83
ε^P	3.16	1.50	0.87	0.20	0.03	0.03	0.14 0.28	
ε^W	0.45	0.39	0.27	0.06	71.08	49.33	45.53	43.52

policy and the estimated rule.

Regarding GDP growth, the Ramsey allocation would have generated more volatility on average than the estimated rule but the cyclical troughs over the last two decades would have been somewhat less pronounced. Examining the level contributions of structural shocks, the contributions of productivity shocks are not strongly different while the contribution of labor supply shocks is much higher with the optimal policy. Consequently, the contribution of efficient supply shocks to GDP growth is significantly positive under the Ramsey allocation in the end-90's while it is slightly negative under the estimated Taylor rule. Similarly, preference shocks have spill-overs on GDP, and therefore contributions, of opposite signs between the Ramsey and the estimated rule: during the period the first half of the 90's, demand shocks are contributing negatively to GDP growth under the estimated rule but bring a strong positive contribution under the Ramsey policy.

Regarding inflation, markup contributions are relatively similar under the Ramsey policy and the estimated rule. Demand shocks however have quasi no impact on inflation with the optimal policy. Regarding efficient supply shocks, on balance, their contributions to inflation have even opposite signs over some sub-sample periods (in particular for the second half of the 90's). This is due to the difference between the transmission mechanisms of labor shocks and productivity shocks under both monetary regimes. With the estimated Taylor rule, productivity and labor supply shocks have similar impact on inflation while the Ramsey policy induces a significantly more muted inflationary effect of labor supply shocks compared with productivity shocks.

Another striking feature of the Ramsey policy regards the interest rate sensitivity to markup shocks. The charts indicate that the optimal policy would have required much stronger reactions of the policy rate to the historical markup shocks than the estimated Taylor rule. In particular, the negative wage-markup shocks recorded from 2002 to 2005 call for lower annual short term interest rate by around 100 bp.

Overall, the analysis of the macroeconomic stabilization properties of the Ramsey policy on the basis of the estimated behaviors and disturbances clearly showed that the typology of efficient and inefficient shocks matters crucially. Unfortunately, the estimation of DSGE models may fail to statistically identify the relative structure of economic disturbances which have dramatically different normative implications. A precise example of such configuration relates to the labor market shocks specified in the model. For the estimation, we introduced a labor supply shock, following an AR(1) process, and a wage markup shocks with an i.i.d. distribution. Without such differences in the stochastic distribution of the shocks, models with only labor supply or wage markup shocks would be observationally equivalent with a first order approximation of the model. However, as we have seen from the impulse response, variance decomposition and counterfactual analysis, the labor supply shocks call for a strong accommodation by the optimal policy resulting in negligible impact on inflation and wages while the wage markup shocks, due to their distortive nature, are allowed to pass-through the nominal side.

5 Optimal Simple rules

The Ramsey allocation obviously constitutes a key normative benchmark to assess the policy implications of the model micro-foundations and the relative role of alternative frictions and structural shocks. Nonetheless, as the size of the model expands, it becomes more difficult but more necessary to streamline the features of optimal stabilization. An approximation of the optimal policy with a simple interest rate rule has generally been considered by the literature as a useful simplification of the optimal behavior. The approach of Giannoni and Woodford [2003a,b] to derive the robust optimal monetary policy rule can in principle be implemented in our DSGE framework using the first order approximation of the first order conditions for the Ramsey problem described in this paper. The initial methodology proposed by the author is normally based on a Linear-Quadratic framework which implies finding the quadratic approximation of the welfare like in Benigno and Woodford [2006] which implies that the optimal rule will only involve target variables. In addition, such rule is robust to the sense that it continues to be optimal regardless of what the structure and the statistical properties of the exogenous disturbances hitting the economy are believed to be. The computation of this approach is beyond the scope of this paper but represents a promising way since it provides a policy rule which exactly implements the Ramsey allocation up to the first order approximation. Note also that this approach can lead to relatively complicated robust optimal rule when using medium to large-scale models, implying that finding more simple optimal rule at the cost of losing robustness would still present some value. In his respect, since our modelling framework is not much more sophisticated that the applications considered by Giannoni and Woodford [2003b], we first intend to specify an interest rate rule inspired from the one they derived within a simplified DSGE framework with mainly price and wage rigidities. We consider an optimal simple optimal rule of the (loglinearized) form:

$$\hat{R}_t = \rho_1 \hat{R}_{t-1} + \rho_2 \hat{R}_{t-2} + r_\pi \hat{\pi}_t + r_{\Delta\pi} \Delta\hat{\pi}_t + r_W \hat{\pi}^w_t + r_{\Delta W} \Delta\hat{\pi}^w_t + r_Y \hat{y}^{gap}_t + r_{\Delta\pi} \Delta\hat{y}^{gap}_t$$

The rule features an AR(2) on the policy rate and reacts to inflation and its first difference, nominal wage inflation and its first difference, as well as model-based output gap and its first difference.

In order to obtain the interest rule approximating the Ramsey allocation, we simulated the model variables under the Ramsey policy given the estimated parameters and the stochastic distribution of structural shocks. Then we estimated the posterior distribution of the coefficients of the interest rate rule using the generated counterfactual data and applying the same estimation techniques and the same set of observed variables than the one used to estimate the benchmark model. We assumed uniform priors on the coefficients of the rule and left unchanged the other structural parameters and the variance of the structural shocks. This approach has the advantage to be much more efficient in terms of optimization and more flexible in terms of rule specification. The best rule is selected using the log data density (LDD) of the simulated dataset and is referred thereafter as ORC (see Table 8). It appears clearly that the design of the optimal simple rule varies significantly with the structure of the shocks present in the economy.

When all the shocks are accounted for, wage inflation turns out to be an important factor shaping the interest rate reaction, compared with more traditional Taylor rules, and enters the rule with a much higher coefficient than the GDP deflator inflation. Another interesting feature of the Taylor Ramsey is the *super inertia* on interest rates. The optimal rule implies not only intrinsic inertia in the dynamics of the interest rate (since a transitory deviation of the inflation rate from its average value increases the interest rate in both the current quarter and the subsequent quarter), but also induces an explosive dynamic for the interest rate if the initial overshooting of the long-run average inflation rate is not offset by a subsequent undershooting (which actually always happens in equilibrium). This "super inertia" property is preserved when changing the structure of the shocks.

However, by removing one after the other all the markup shocks from the disturbances set, the coefficients of the optimal rule are sensibly modified. In particular, the coefficient on the first difference term of the output gap increases as the inefficient shocks are removed and the presence of first difference terms strongly improve the *performance* of the simple optimal rule, in the sense of the marginal density. This illustrates again the need to derive optimal simple interest rate rules which are robust to the structure of economic shocks.

Tab. 8 - POSTERIOR PARAMETERS OF ORC RULES WITH DIFFERENT SET OF SHOCKS

Shocks	r_π	$r_{\Delta\pi}$	r_w	$r_{\Delta w}$	r_y	$r_{\Delta y}$	ρ_1	ρ_2	LDD
All									
ORC1	0.263	-	0.959	-	-	-	1.637	-0.738	-393.031
ORC2	0.252	-	1.243	-	0.102	-	1.942	-0.865	-381.520
ORC3	0.090	0.139	1.399	-0.167	0.127	-0.053	1.909	-0.766	-390.682
No ε^W									
ORC1	0.520	-	0.948	-	-	-	1.219	-0.422	-120.589
ORC2	0.357	-	1.191	-	0.064	-	1.279	-0.418	-78.432
ORC3	0.821	0.106	1.214	-0.112	-0.001	0.779	2.002	-1.022	-16.025
No ε^W,ε^Q									
ORC1	0.261	-	0.429	-	-	-	1.463	-0.599	77.984
ORC2	0.191	-	0.538	-	0.030	-	1.447	-0.554	94.021
ORC3	0.723	0.109	0.758	-0.041	-0.057	0.941	2.175	-1.190	225.247
Efficient									
ORC1	1.493	-	1.715	-	-	-	1.672	-0.733	425.895
ORC2	1.428	-	1.692	-	0.010	-	1.668	-0.723	420.112
ORC3	1.425	-2.563	1.767	0.854	0.008	1.011	2.334	-1.400	485.242

The literature on optimal monetary policy has extensively explored the alternative ways to implement the Ramsey equilibrium using simple feedback interest rate rules. Schmitt-Grohe and Uribe [2005] notably propose to focus on finding parameterizations of interest rate rules with the following features: involvement of only few observable macroeconomic variables, local uniqueness of the rational expectation equilibrium, no violation of the zero bound constraint (imposing that the steady state nominal interest rate is higher than twice its standard deviation), and maximization of the welfare conditional on the deterministic steady state of the Ramsey economy. Such concept of optimal operational rule has been implemented in our framework using the a second-order numerical approximation of the conditional welfare.

Tab. 9 - OWB RULES

Rule	ρ	r_π	r_{π_w}	WelfareCost
1	0.947	0.769	1.6922	-1.474
2	0.615	2	-	-1.403
3	1	-	2.076	-1.467
No ZLB constraint	0.816	1.494	4	-1.478
No inefficient shocks	1.111	4	3.7895	-1.445

However, since the optimization procedure is relatively time consuming, we restrained the interest rule to the form:

$$\hat{R}_t = \rho \hat{R}_{t-1} + r_\pi \hat{\pi}_t + r_W \hat{\pi}_t^w$$

The best rule has significant interest rate smoothing and reacts both to price inflation and wage inflation (see Table 7). The preferred optimal operational rule within the class considered here, has a higher weight on nominal wage growth than on inflation and features a relatively high degree of interest rate smoothing. Those features are even more pronounced when the constraint on the zero lower bound is relaxed. Thereafter, we refer to this rule as the optimal welfare-based rule (OWB). Schmitt-Grohe and Uribe [2005] argue that the benefits of interest rate smoothing are limited in terms of welfare and that simple rules responding aggressively to price inflation already represent a good approximation of the Ramsey policy. Nonetheless, within their framework which differs from ours in particular on the shock structure and on the micro-foundation of the labor market frictions, higher wage rigidity leads to an optimal rule with a strong relative weight on wage inflation and *superinertial* response to lagged interest rate. As pointed out by the authors, the micro-foundations of the labor market nominal frictions are different from the one introduced in our model. In particular, for a same estimated elasticity of wage to the marginal rate of substitution between leisure and consumption, the implied *Calvo-type* rigidities in Schmitt-Grohe and Uribe [2005] would be higher than in our case. Therefore, given the estimated parameter for wage rigidity, our optimal rule computations compare more directly to the case of *high wage stickiness* exposed the authors. Regarding the sensitivity of the optimal operational rule to the structure of the shocks, the coefficients of the rule obtained when only the efficient shocks are introduced, change significantly. This once again indicate that such simple rule are not robust in the sense of Giannoni and Woodford [2003a].

In the following, we examine the performance of both the OWB and the ORC rules in approximating the Ramsey allocation through different dimensions: comparison of the welfare (see Table 9), counterfactual analysis (see Figure 23) and impulse response analysis (see Figures 15 to 22). Broadly speaking, the two simple rules deliver relatively similar allocations than the Ramsey policy. The more pronounced differences are related the dynamics of the interest rate and the transmission of markup shocks.

6 Conclusion

In this paper, we have built on the literature estimating DSGEs in order to explore within a more operational framework, the normative prescriptions of such structural models regarding the optimal conduct of monetary policy over the business cycle. We find that:

(i) The Ramsey policy is not operational in the sense that it induces a high probability to tilt the zero bound. A more striking result is the negligible welfare cost of imposing the zero lower bound, meaning that even if the volatility of the policy instrument is highly constrained, monetary policy is still effective in improving the welfare of agents.

(ii) We highlight the need to improve the economic micro-foundation and the econometric identification of the structural disturbances when bringing together estimated models and optimal policy analysis. In particular, we show that efficient labor supply shocks and inefficient wage markup shocks are close to observationally from the empirical perspective while they have crucially different implications for optimal policy. The labor supply shocks is indeed fully accommodated in the Ramsey allocation whereas the wage markup shocks are fully allowed to *pass-through* wage and price dynamics.

(iii) The preceding point is crucial when looking for simple rules approximating the optimal policy which are very sensitive to the structure of economic shocks.

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Fig. 3 - IMPULSE RESPONSES TO A TECHNOLOGY SHOCK. Ramsey (red lines), estimated Rule (Blue lines), 90 percent highest probability density intervals (doted lines).

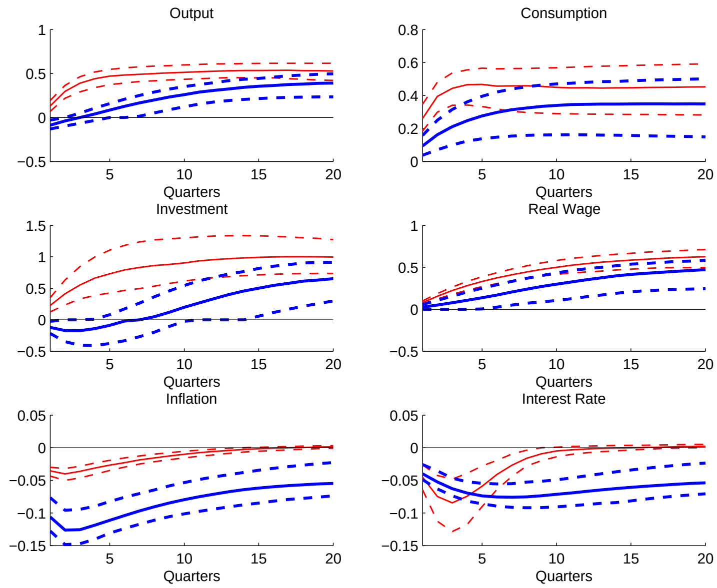

Fig. 4 - IMPULSE RESPONSES TO A PREFERENCE SHOCK. Ramsey (red lines), estimated Rule (Blue lines), 90 percent highest probability density intervals (doted lines).

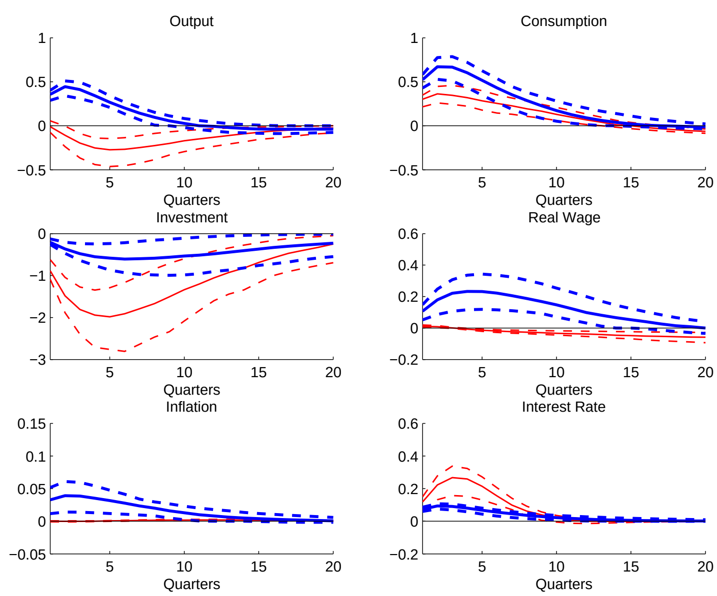

Fig. 5 - IMPULSE RESPONSES TO A GOVERNMENT SPENDING SHOCK. Ramsey (red lines), Estimated (blue lines), 90 percent highest probability density intervals (doted lines).

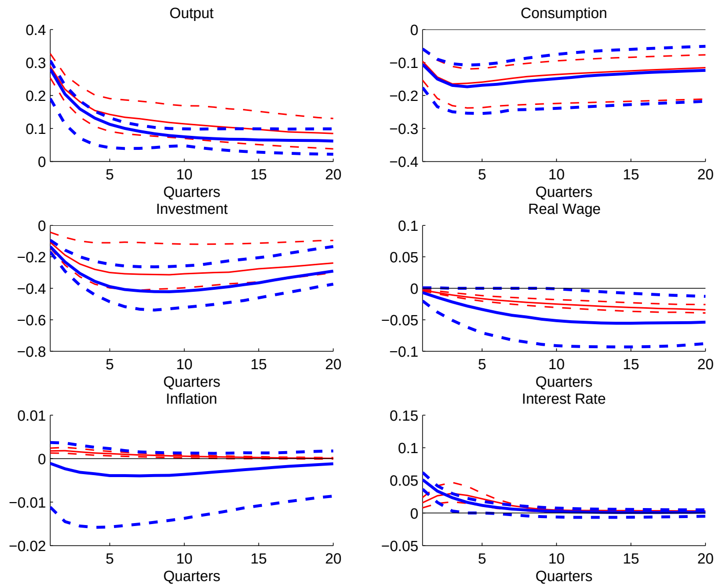

Fig. 6 - IMPULSE RESPONSES TO AN INVESTMENT SHOCK. Ramsey (red lines), Estimated (blue lines), 90 percent highest probability density intervals (doted lines).

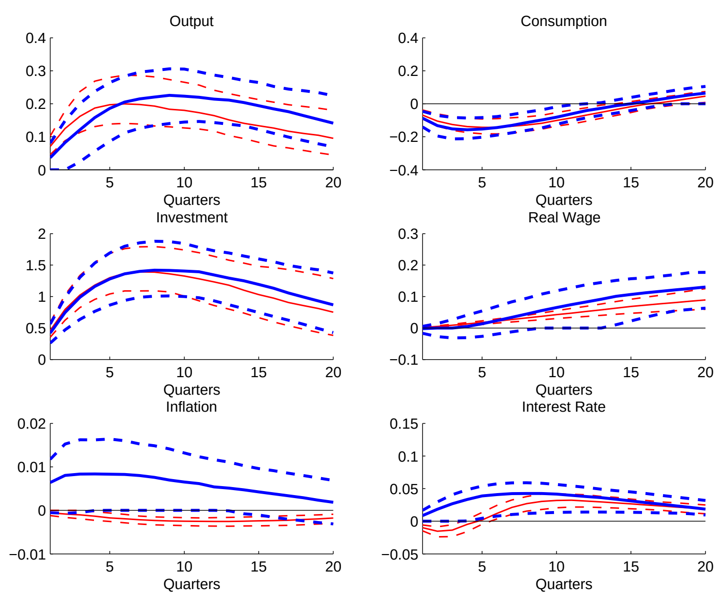

Fig. 7 - Impulse Responses to a Labour Supply Shock. Ramsey (red lines), Estimated (blue lines), 90 percent highest probability density intervals (doted lines).

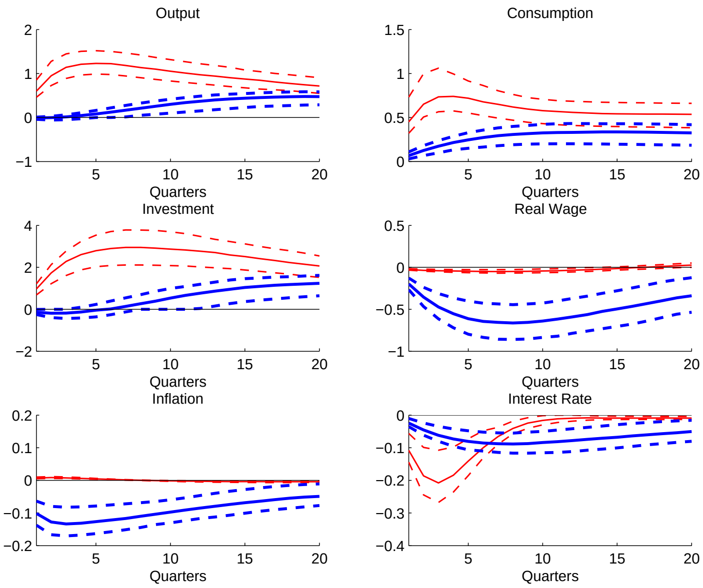

Fig. 8 - Impulse Responses to a Price Markup Shock. Ramsey (red lines), Estimated (blue lines), 90 percent highest probability density intervals (doted lines).

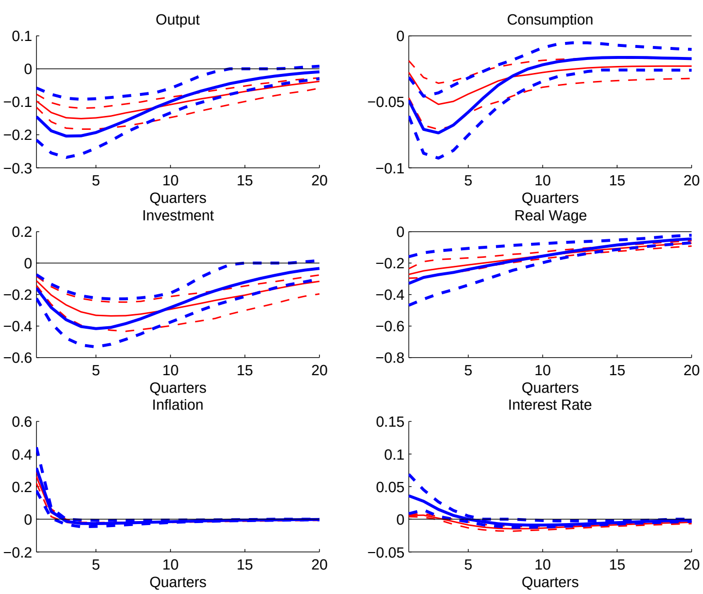

Fig. 9 - IMPULSE RESPONSES TO AN EXTERNAL FINANCE PREMIUM SHOCK. Ramsey (red lines), Estimated (blue lines), 90 percent highest probability density intervals (doted lines).

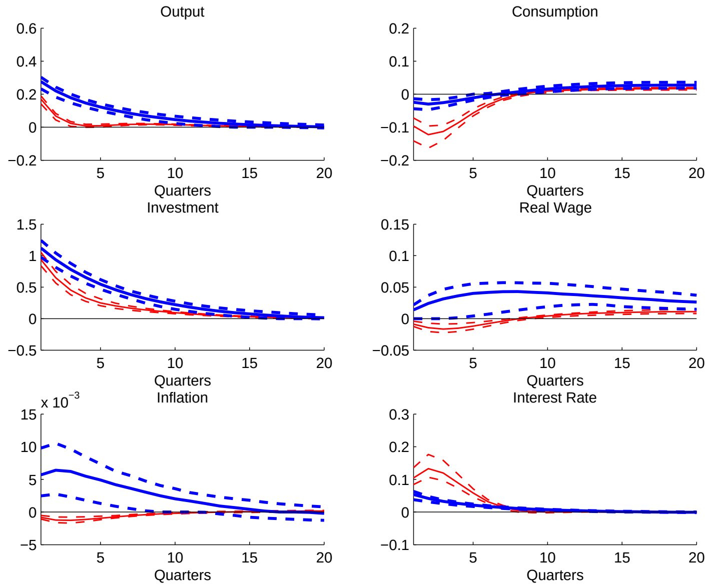

Fig. 10 - IMPULSE RESPONSES TO A WAGE MARKUP SHOCK. Ramsey (red lines), Estimated (blue lines), 90 percent highest probability density intervals (doted lines).

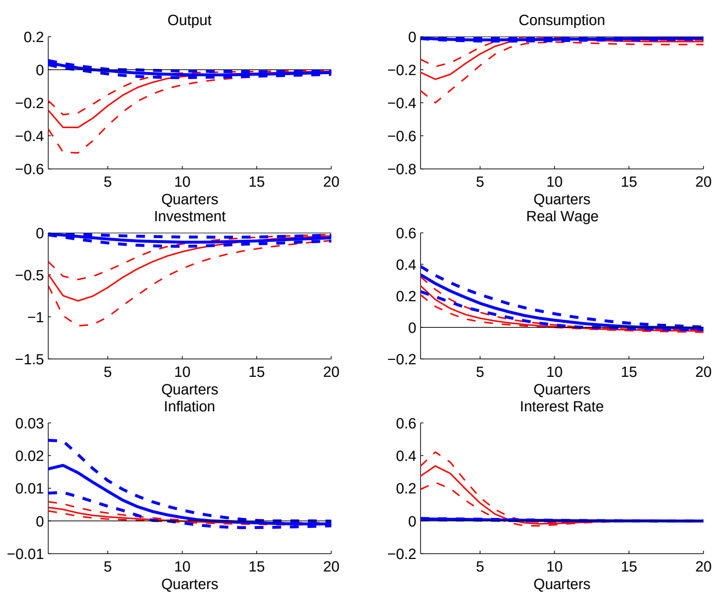

Fig. 11 - COUNTERFACTUAL

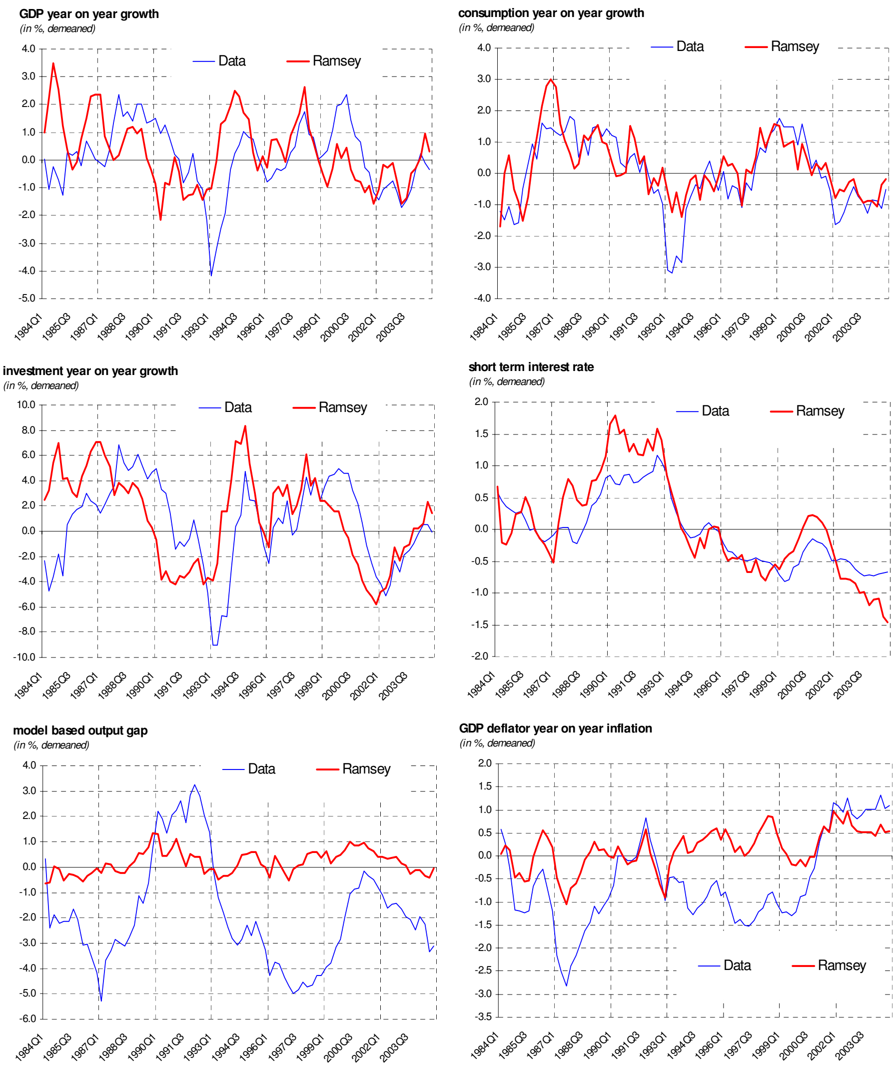

Fig. 12 - Comparison of Counterfactuals, Contributions to GDP Year on Year Growth.

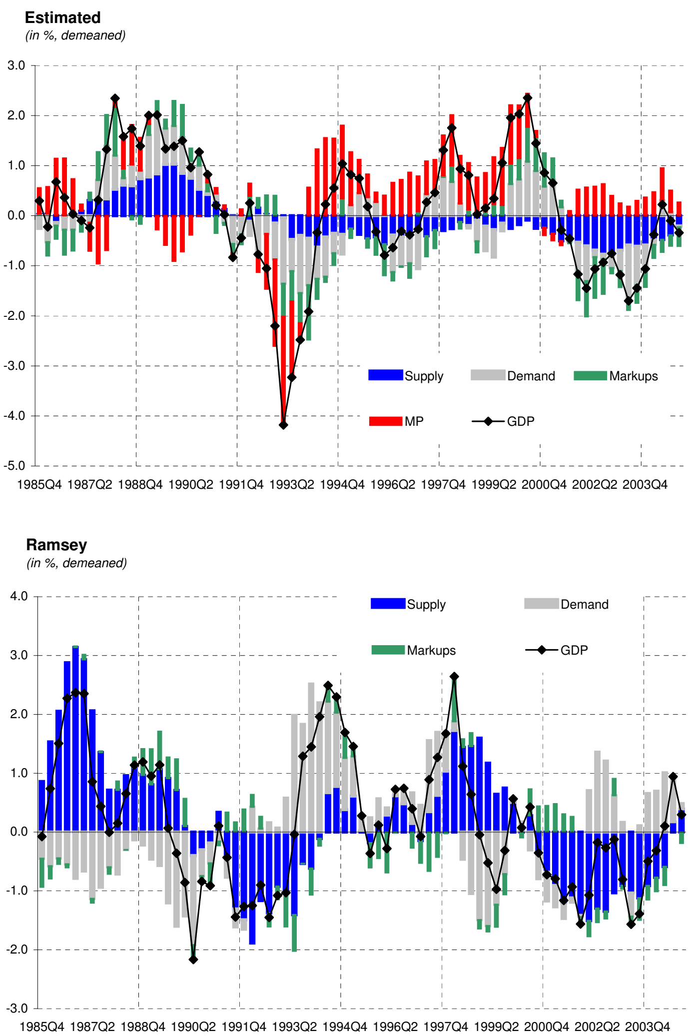

Fig. 13 - COMPARISON OF COUNTERFACTUALS, CONTRIBUTIONS TO GDP DEFLATOR YEAR ON YEAR INFLATION.

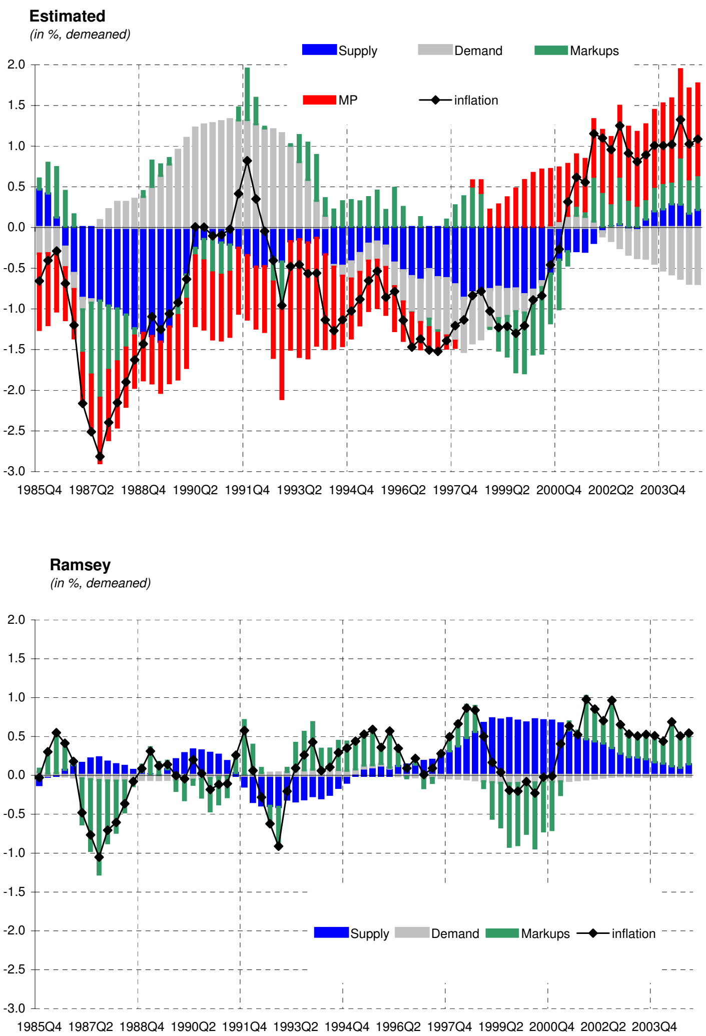

Fig. 14 - Comparison of Counterfactuals, Contributions to Short Term Interest Rate.

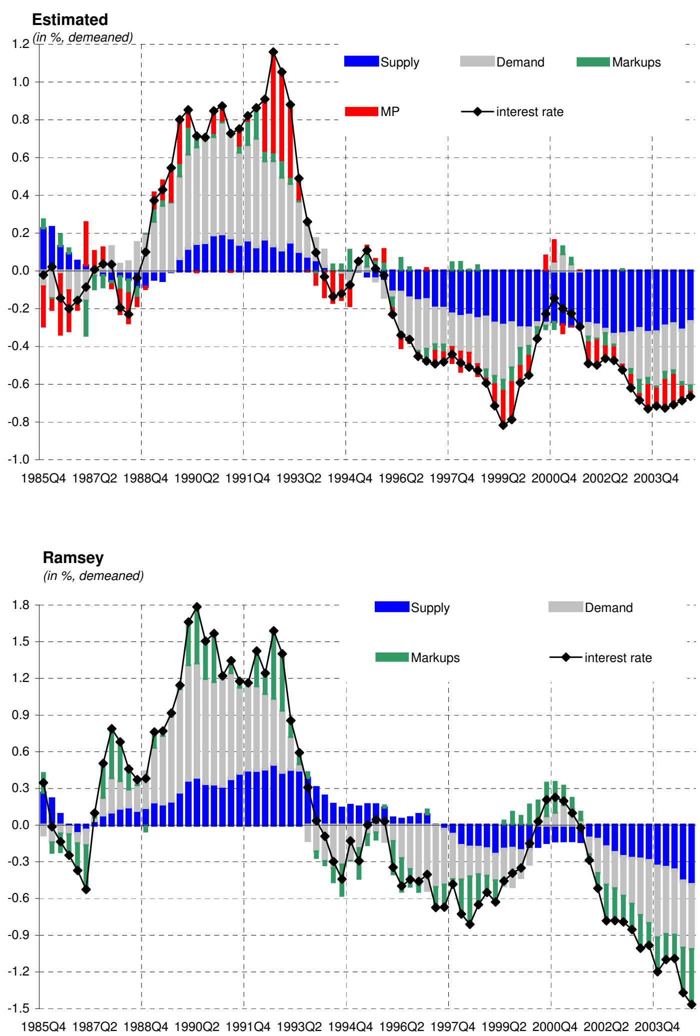

Fig. 15 - IMPULSE RESPONSES TO A TECHNOLOGY SHOCK. Ramsey (red lines), OWB (Blue lines), ORC (Green lines).

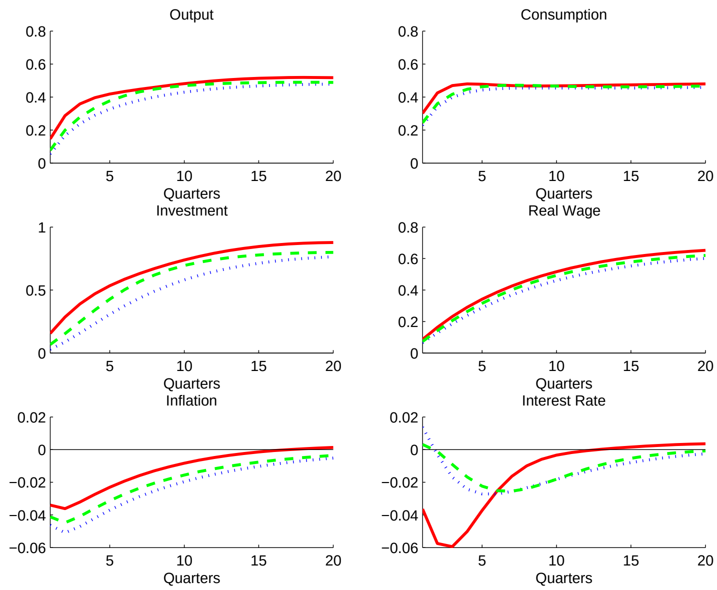

Fig. 16 - IMPULSE RESPONSES TO A PREFERENCE SHOCK. Ramsey (red lines), OWB (Blue lines), ORC (Green lines).

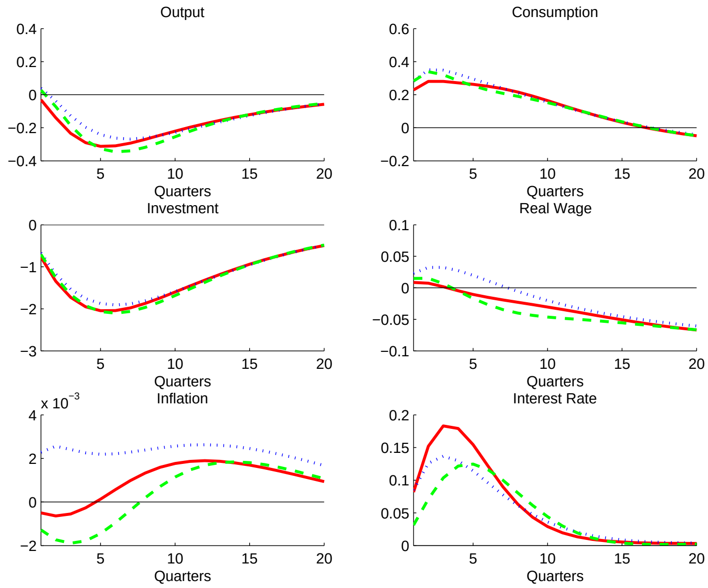

Fig. 17 - IMPULSE RESPONSES TO A PUBLIC EXPENDITURE SHOCK. Ramsey (red lines), OWB (Blue lines), ORC (Green lines).

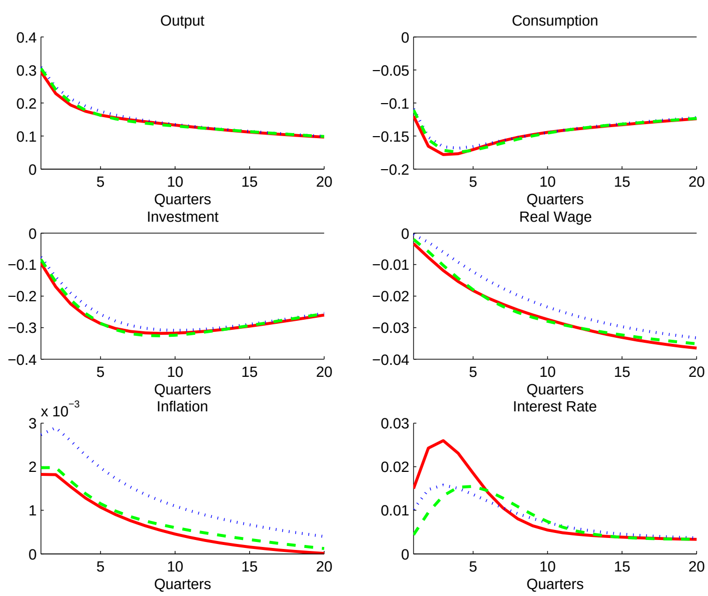

Fig. 18 - IMPULSE RESPONSES TO AN INVESTMENT SHOCK. Ramsey (red lines), OWB (Blue lines), ORC (Green lines).

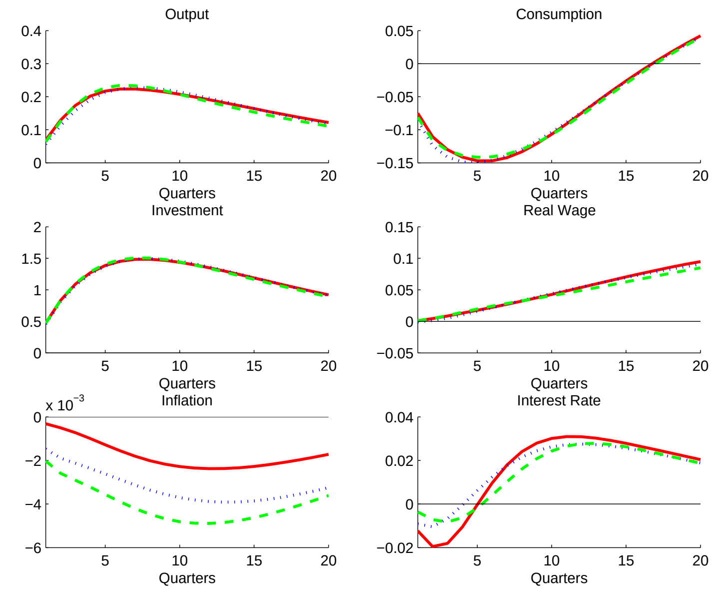

Fig. 19 - IMPULSE RESPONSES TO A LABOUR SUPPLY SHOCK. Ramsey (red lines), OWB (Blue lines), ORC (Green lines).

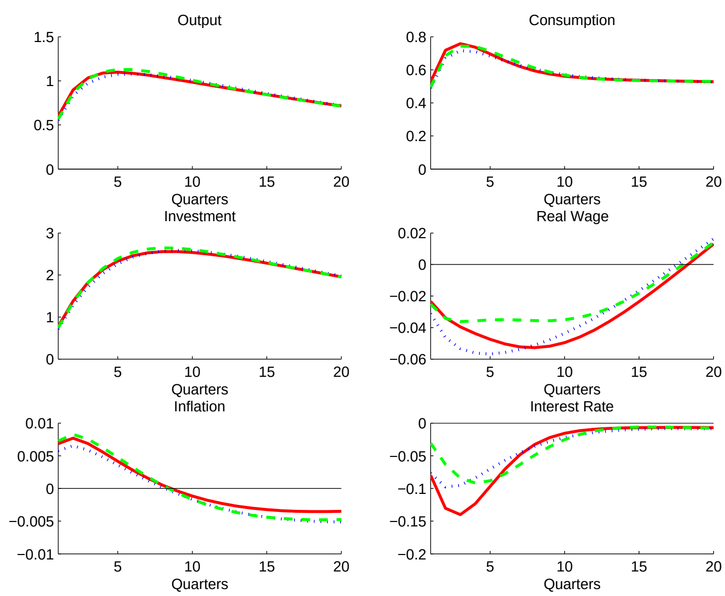

Fig. 20 - Impulse Responses to a Price Markup Shock. Ramsey (red lines), OWB (Blue lines), ORC (Green lines).

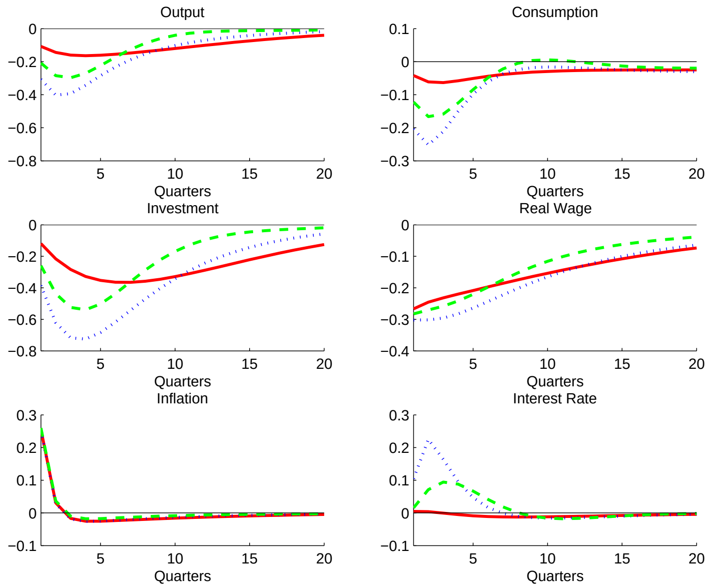

Fig. 21 - Impulse Responses to an External Finance Risk Premium Shock. Ramsey (red lines), OWB (Blue lines), ORC (Green lines).

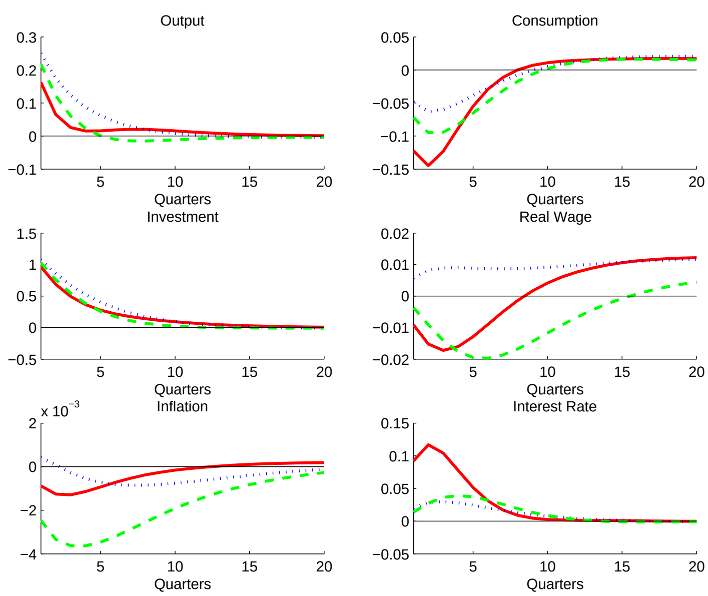

Fig. 22 - IMPULSE RESPONSES TO A WAGE MARKUP SHOCK. Ramsey (red lines), OWB (Blue lines), ORC (Green lines).

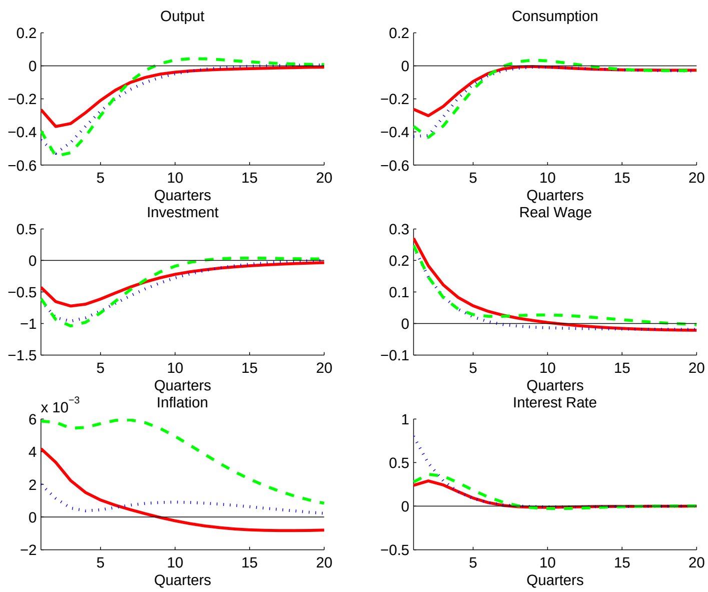

Fig. 23 - Comparison of Ramsey, ORC and OWB Counterfactuals